#### DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

#### COMBINED COMPETITIVE (PRELIMINARY) EXAMINATION, 2013

# MATHEMATICS Code No. 13



Time Allowed: Two Hours

Maximum Marks: 300

#### **INSTRUCTIONS**

- 1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
- 2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C OR D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE RESPONSE SHEET.
- You have to enter your Roll Number on this
   Test Booklet in the Box provided alongside.
   DO NOT write anything else on the Test Booklet.

Your Roll No.	

- 4. This Booklet contains 100 items (questions). Each item comprises *four* responses (answers). You will select *one* response which you want to mark on the Response Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.
- 5. In case you find any discrepancy in this test booklet in any question(s) or the Responses, a written representation explaining the details of such alleged discrepancy, be submitted within three days, indicating the Question No(s) and the Test Booklet Series, in which the discrepancy is alleged. Representation not received within time shall not be entertained at all.
- 6. You have to mark all your responses ONLY on the separate Response Sheet provided. *See directions in the Response Sheet*.
- 7. All items carry equal marks. Attempt ALL items. Your total marks will depend only on the number of correct responses marked by you in the Response Sheet.
- 8. Before you proceed to mark in the Response Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Response Sheet as per instructions sent to you with your Admit Card and Instructions.
- 9. While writing Centre, Subject and Roll No. on the top of the Response Sheet in appropriate boxes use "ONLY BALL POINT PEN".
- 10. After you have completed filling in all your responses on the Response Sheet and the examination has concluded, you should hand over to the Invigilator only the Response Sheet. You are permitted to take away with you the Test Booklet.

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	1.	If $A = \{x, y \mid x^2 + y^2 = 25\}$ and $B = \{x, y \mid x^2 + (A) \text{ One point }$ (C) Three points	(B)	144} then A∩B contains: Two points Four points		
	2. The number of subsets of a set containing n elements is:					
		(A) n		$2^{n} - 1$		
		(C) $n^2$	(D)	$2^{n}$		
	<ul> <li>3. 20 teachers of a school either teach Maths or Physics. 12 of them teach Maths while 4 to both the subjects. The number of teachers teaching Physics only is:</li> <li>(A) 12</li> <li>(B) 8</li> </ul>				e 4 teach	
		(C) 16	` ′	None of these		
		(2) 10	(D)	None of these		
	4. If a relation R is defined on the set Z of integers as follows:				. Then	
		Domain(R) =	(D)	(0, 2, 4, 5)		
		(A) {3, 4, 5}		$\{0, 3, 4, 5\}$		
		(C) $\{0, \pm 3, \pm 4, \pm 5\}$	(D)	None of these		
	5.	If R is a relation on a finite set having n elements,	then th	ne number of relations on A i	s:	
		(A) $2^{n}$	(B)			
		(C) $n^2$	(D)	$n^n$		
<b>88</b> 2 <b>8</b> 3√€	6. <b>R</b> €	R is a relation on the set Z of integers and it is given $(X) = 1.315 \overline{Z}_{1} + 5 \overline{Z}_{1} = 0$	en by		nRis:	
<del>-</del> -,-,				Reflexive and Symmetric		
		(C) Symmetric and Transitive	(D)	An equivalence relation		
	7.	The equation	repres	sents a circle of radius:		
		(A) 5	(B)	$2\sqrt{5}$		
		(C) $\frac{5}{2}$	(D)	None of these		
		If $Z_1$ , $Z_2$ , $Z_3$ are complex numbers such that :				
		$ Z_1  =  Z_2  =  Z_3  = \left  \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right  = 1 \text{ then }  Z_1 + Z_2  = 1$	$\mathbf{Z}_2 + \mathbf{Z}_2$	3 is:		
		(A) Equal to 1	(B)	Less than 1		
		(C) Greater than 1	` ′	Equal to 3		
				-		
	9.	The locus of point Z satisfying $Re(Z^2) = 0$ is:				
	9.	(A) A pair of straight lines	` ′	A circle		
	9.	· · · · · · · · · · · · · · · · · · ·	` ′	A circle None of these		

10. If  $Z_r = \cos\left(\frac{2r\pi}{5}\right) + i\sin\left(\frac{2r\pi}{5}\right)$ , r = 0,1,2,3,4 then  $Z_0 \times Z_1 \times Z_2 \times Z_3 \times Z_1$ 

(A) -1

(B) 0

(C) 1

(D) None of these

11. If  $\alpha$ ,  $\beta$ ,  $\gamma$  are the roots of the equation  $x^3 + 4x + 1 = 0$ . Then  $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1} =$ 

(A) 2

(B) 3

(C) 4

(D) 5

12. Let A, G and H be the Arithmetic mean, Geometric mean and Harmonic mean of two positive numbers a and b. The quadratic equation whose roots are A and H is:

- (A)  $Ax^2 (A^2 + G^2)x + AG^2 = 0$
- (B)  $Ax^2 (A^2 + H^2)x + AH^2 = 0$
- (C)  $Hx^2 (H^2 + G^2)x + HG^2 = 0$
- (D) None of these

13. G is a group under  $\otimes_7$  where G = {1, 2, 3, 4, 5, 6}. If  $5 \otimes_7 x = 4$  then x =

(A) 0.8

(B) 4

(C) 3

(D) 5

14. In the group  $G = \{1, 3, 7, 9\}$  under multiplication module  $\{10, (3 \times 7^{-1})^{-1}\}$  is equal to :

(A) 9

(B) 5

(C) 7

(D) 3

15. The identity element in the group  $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} \middle| x \neq 0 \text{ and } x \text{ is real} \right\}$  with respect to matrix multiplication is:

(A)  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ 

(B)  $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ 

(C)  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 

(D) None of these

16. If  $a * b = a^2 + b^2$ , then the value of (4\*5)\*3 is:

(A)  $(4^2 + 5^2) + 3^2$ 

(B)  $(4+5)^2+3^2$ 

(C)  $41^2 + 3^2$ 

(D)  $(4+5+3)^2$ 

17.	17. In Z, the set of all integers, the inverse of -7 with respect to defined by all $a, b \in Z$ is:		or		
		(D) 7			
	(A) -14	(B) 7			
	(C) -7	(D) None of these			
18.	The units of the field $F = \{0, 2, 4, 6, 8\}$ under	are:			
	(A) $\{0\}$	(B) $\{2, 4, 6, 8\}$			
	(C) F	(D) None of these			
19.	$(Z_n, \oplus_n, \otimes_n)$ is a field if and only if n is:				
	(A) Even	(B) Odd			
	(C) Prime	(D) None of these			
		<b>,</b>			
20.	The ideals of a field F are:				
	(A) Only {0}	(B) Only F			
	(C) Both $\{0\}$ and F	(D) None of these			
21.	Every finite integral domain is:				
	(A) Not a field	(B) Field			
	(C) Vector space	(D) None of these			
22.	22. The order of i in the multiplicative group of fourth roots of unity is:				
	(A) 4	(B) 3			
<b>D</b> pb <del>an</del> Da⊗ <sub>1</sub> b	o(C) 2	(D) 1			
23.	23. The non–zero elements a, b of a ring $(R, +, .)$ are called zero divisors if:				
	(A) $a.b=0$	(B)			
	(C)	(D)			
24.	If the ring R is an integral domain then:				
	(A) $R[x]$ is a field	(B) R[x] is an integral domain			
	(C) R[x] is not an integral domain	(D) None of these			
25.	The product of an even permutation and an odd pe	ermutation is :			
	(A) Even	(B) Odd			
	(C) Neither ever nor odd	(D) None of these			

26. If :

 $(A) \ \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ 

 $(B) \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 

(C)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 

(D) None of the above

27. If AB = A and BA = B where A and B are square matrices then:

(A)  $A^2 = A$  and  $B^2 = B$ 

(B)  $A^2 \neq A$  and  $B^2 = B$ 

(C)  $A^2 = A$  and  $B^2 \neq B$ 

(D)  $A^2 \neq A$  and  $B^2 \neq B$ 

28. If  $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ , then the value of |adj A| is :

(A)  $a^{27}$ 

(B)  $a^9$ 

(C) a<sup>6</sup>

(D) a<sup>2</sup>

29. If  $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ , then |adj (adj A)| is:

(A)  $14^4$ 

(B)  $14^3$ 

(C)  $14^2$ 

(D) 14

30. If  $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ , and  $A^T + A = I_2$  where  $A^T$  is the transpose of A and  $I_2$  is the 2×2 Unit matrix.

Then:

(A)  $\theta = n \pi, n \in \mathbb{Z}$ 

(B)

(C)  $\theta = 2n\pi + \frac{\pi}{3}, n \in Z$ 

(D) None of these

- 31. The matrix  $A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$  is nilpotent of index:
  - (A) 2

(B) 3

(C) 4

- (D) None of these
- 32. The rank of the matrix  $A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$  is :
  - (A) 2

**(B)** 3

(C) 1

- (D) Indeterminate
- 33. For what value of  $\lambda$ , the system of equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

 $x + 2y + \lambda z = 12$  is Inconsistent?

(A) 
$$\lambda = 1$$

(B) 
$$\lambda = 2$$

(C) 
$$\lambda = -2$$

(D) 
$$\lambda = 3$$

- 34. If A is a  $3\times3$  matrix and B is its adjoint such that |B| = 64, then |A| =
  - (A) 64

(B) 
$$\pm 64$$

, then 
$$(\mathbb{G}) A + A^2 + A^3 + \dots \infty$$

- (D) 18
- 35. If  $A^3 = 0$ , then  $1 + A + A^2$  equals:
  - (A) 1 A

(B)  $(1 - A)^{-1}$ 

(C)  $(1 + A)^{-1}$ 

(D) None of these

36. If A =

#### equals to:

$$(A) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(B) 
$$\begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

(C) 
$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{2} & 0 \end{bmatrix}$$

$$(D) \begin{bmatrix} -\frac{1}{4} & \frac{1}{3} \\ \frac{1}{2} & 0 \end{bmatrix}$$

37. If s = a + b + c then the value of  $\Delta = \begin{vmatrix} s+c & a & b \\ c & s+a & b \\ c & a & s+b \end{vmatrix}$  is:

(A)  $2s^2$ 

(B)  $2s^3$ 

(C)  $s^3$ 

(D)  $3s^3$ 

38.  $\lim_{n \to \infty} \left[ \frac{4^{\frac{1}{n}} - 1}{\frac{1}{3^{\frac{1}{n}} - 1}} \right]$  is equal to:

(A)  $\log_4 3$ 

(B) log<sub>3</sub> 4

(C) 1

(D) None of these

39. The value of  $\lim_{n \to \infty} \left[ \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n+1)(2n+3)} \right]$  is:

(A) 1

(B)  $\frac{1}{2}$ 

(C)  $-\frac{1}{2}$ 

(D) None of these

 $40. \quad \lim_{x \to \infty} \left[ \frac{\int_0^{2x} x e^{x^2} dx}{e^{4x^2}} \right] =$ 

(A) 0

(B) ∞

(C) 2

(D)

41. The function  $f(x) = \begin{cases} 1 - 2x + 3x^2 - 4x^3 + \dots + \infty & \text{if } x \neq -1 \\ 1 & \text{if } x = -1 \end{cases}$  is:

- (A) Continuous and differentiable at x = -1
- (B) Neither continuous nor differentiable at x = -1
- (C) Continuous but not differentiable at x = -1
- (D) None of the above

42. Let 
$$f(x) = \begin{cases} \frac{\sin \pi x}{5x} &, x \neq 0 \\ K &, x = 0. \end{cases}$$

If f(x) is continuous at x = 0, then the value of K is:

(A)  $\frac{\pi}{5}$ 

(B)

(C) 1

(D) 0

43. If f(x) is differentiable and strictly increasing function, then the value of  $\lim_{x\to 0} \left\lceil \frac{f(x^2) - f(x)}{f(x) - f(0)} \right\rceil$  is:

(A) 1

(B) 0

(C) -1

(D) 2

44. The number of points at which the function f(x) = |x-3| + |x+1| does not have a derivative in the interval [-4, 4] is:

(A) 1

(B) 2

(C) 3

(D) None of these

45. If f(x) satisfies the conditions of Rolle's theorem in [1, 2] and f(x) is continuous in [1, 2], then  $\int_{0}^{2} f'(x) dx$  is equal to:

 $\int_{A}^{A} (x) dx$ (A) 3

(B) 0

(C) 1

(D) 2

46. Let  $f(x) = e^x$ ,  $x \in [0,1]$ , then a number 'c' of the Lagrange's mean value theorem is :

(A)  $\log_e(e-1)$ 

(B)  $\log_e(e+1)$ 

(C) 1

(D) None of these

47. The maximum value of xy subject to x + y = 8 is:

(A) 8

(B) 16

(C) 20

(D) 24

48. The series  $n - \frac{n^2}{2} + \frac{n^3}{3} - \frac{n^4}{4} + -+ \dots -1 < n \le 1$  represents the function:

(A) sin n

(B) cos n

(C)  $(1+n)^n$ 

(D)  $\log (1 + n)$ 

49. Expansion of  $\sin x$  in powers of  $\left(x - \frac{\pi}{2}\right)$  is :

(A) 
$$\left(x - \frac{\pi}{2}\right) - \frac{\left(x - \frac{\pi}{2}\right)^3}{2 + \frac{1}{2}} + \frac{\left(x - \frac{\pi}{2}\right)^5}{2 + \frac{1}{2}} - + \dots$$

(B) 
$$\left(x - \frac{\pi}{2}\right) + \frac{\left(x - \frac{\pi}{2}\right)^3}{\underline{3}} + \frac{\left(x - \frac{\pi}{2}\right)^5}{\underline{5}} + \dots$$

(C) 
$$1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{\underline{|2|}} + \frac{\left(x - \frac{\pi}{2}\right)^4}{\underline{|4|}} - + \dots$$

- (D) None of these
- 50. The equation of tangent to the curve  $x = t^3 4$ ,  $y = 2t^2 + 1$  at the point where t = 2 is:

(A) 
$$2x - 3y - 19 = 0$$

(B) 
$$2x - 3y + 19 = 0$$

(C) 
$$2x + 3y - 19 = 0$$

(D) 
$$3x + 2y + 6 = 0$$

51. If the normal to the curve  $y^2 = 5x - 1$  at the point (1, -2) is of the form ax - 5y + b = 0. Then 'a' and 'b' are:

$$(C)$$
 -4, 14

(D) 
$$-4, -14$$

52. The least value of  $f(x) = 2x + \frac{8}{x^2}$ , x > 0 is:

$$(A)$$
 4

$$(C)$$
 8

53. The radius of curvature for the curve  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2b^2}$  is:

(A) 
$$\frac{p^2}{a^2b^2}$$

(B) 
$$\frac{a^2 p^2}{b^2}$$

(C) 
$$\frac{a^2b^2}{p^3}$$

(D) 
$$a^2 b^2 p^2$$

(A) 
$$\left(0,\frac{1}{2}\right)$$

(B) 
$$\left(\frac{1}{2}, \frac{1}{2}\right)$$

(C) 
$$\left(\frac{1}{2},0\right)$$

(D) None of these

55. The radius of curvature of the curve  $r = a \sin n \theta$  at origin is:

(B)

(D)  $\frac{2na}{3}$ 

56. The asymptote parallel to co-ordinate axes of the curve  $(x^2 + y^2) x - ay^2 = 0$  is:

(A) 
$$y - a = 0$$

(B) 
$$y + a = 0$$

(C) 
$$x - a = 0$$

(D) 
$$x + a = 0$$

57. The asymptote of the curve  $y = e^x$  is given by:

(A) 
$$y = 0$$

(B) 
$$x = 0$$

(C) 
$$y = e$$

(D) 
$$x = e$$

58. For the curve  $y^2(1+x) = x^2(1-x)$ , the origin is a:

(A) Node

2

(B) Cusp

(C) Conjugate point

(D) None of these

59. The curve  $y = x^3 - 3x^2 - 9x + 9$  has a point of inflexion at:

(A) x = -1

(B) x = 1

(C) x = -3

(D) x = 3

60. The curve  $y = \log x$  is:

- (A) Concave upwards in  $(0, \infty)$
- (B) Concave downwards in  $(0, \infty)$
- (C) Concave upwards in  $(-\infty, \infty)$
- (D) Concave downwards in  $(-\infty, \infty)$

61. The points of inflexion on the curve  $x = (\log y)^3$  are:

(A) (0, 1) and  $(8, e^2)$ 

(B) (1, 0) and  $(8, e^2)$ 

(C) (0, 1) and  $(e^2, 8)$ 

(D) (1, 0) and  $(e^2, 8)$ 

62. The graph of  $x = \frac{1-t^2}{1+t^2}$ ,  $y = \frac{2t}{1+t^2}$  is a:

(A) Circle

(B) Ellipse

(C) Cycloid

(D) None of these

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11

[Turn over

- 63. The number of leaves in the curve  $r = a \sin 5\theta$  are :
  - (A) Two

(B) Five

(C) Ten

- (D) None of these
- 64. If  $u = f(y+ax) + \phi(y-ax)$  then  $\frac{\partial^2 u}{\partial x^2} =$ 
  - (A)  $\frac{\partial^2 u}{\partial v^2}$

(B)  $a^2 \frac{\partial^2 u}{\partial y^2}$ 

(C)  $-a^2 \frac{\partial^2 u}{\partial y^2}$ 

- (D) a  $\frac{\partial^2 \mathbf{u}}{\partial \mathbf{y}^2}$
- 65. If  $Z = \log (x^2 + y^2)$  then  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} =$ 
  - (A) 0

(B) 1

(C) 2

- (D) 3
- 66. If  $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots + \infty}}}$  then  $(2y-1)\frac{dy}{dx}$  is given by:
  - (A) sin x

(B) cos x

(C) tan x

- (D) cot x
- 67. The series  $1 \frac{1}{2} + \frac{1}{4} \frac{1}{8} + \frac{1}{16} + \dots$  is:
  - (A) Conditionally Convergent

(B) Absolutely Convergent

(C) Divergent

- (D) None of the above
- 68. The series  $1 \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \frac{1}{\sqrt{4}} + \dots$  is:
  - (A) Conditionally Convergent

(B) Absolutely Convergent

(C) Oscillatory

(D) None of the above

- 69. The series  $\sum_{n=1}^{\infty} \frac{(n-2\log n)^n}{2^n \ n^n} \ is:$ 
  - (A) Convergent

(B) Divergent

(C) Oscillatory

(D) None of these

- 70. The series  $\sum_{n=1}^{\infty} \frac{\lfloor n \rfloor 2^n}{n^n}$  is:
  - (A) Convergent

(B) Divergent

(C) Oscillatory

- (D) None of these
- 71. The series  $\sum_{n=1}^{\infty} \frac{4 \cdot 7 \cdot .....(3x+1)}{1 \cdot 2 \cdot ......x} x^n$  is Convergent if :
  - (A) |x| < 1

(B)

(C)  $|x| < \frac{1}{4}$ 

(D)  $|x| < \frac{1}{2}$ 

- 72.  $\int^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx =$ 
  - (A) 0

(B)  $\frac{1}{2}$ 

(C) 1

(D) None of these

73. 
$$\int_{0}^{\pi} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx =$$

$$\lim_{x \to \infty} \left[ \frac{1}{3_{n+1}} + \frac{1}{(2n)^{2}} \pm \frac{1}{n+3} + \dots + \frac{1}{2n} \right] =$$

(B)

(C)

(D)

- 74.
- (A) log<sub>e</sub> 2

(B)  $\log_e 3$ 

(C) log<sub>e</sub> 6

- (D) None of these
- 75. The entire length of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$  is:
  - (A) 8a

(B)  $4\sqrt{3}a$ 

(C) 6a

(D)  $\sqrt{8a}$ 

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13

76.	The perimeter of $r = a (1 + \cos \theta)$ is:		
	(A) a	(B)	2a
	(C) 4a	(D)	8a
77.	The length of one arch of Cycloid $n = a(\theta + \sin\theta)$	y = a	$a(1-\cos\theta)$ is:
	(A) a	(B)	4a
	(C) 8a	(D)	32a
78.	The area bounded by the curve $y = 2x$ , $\dot{x} - axis$ ar	ıd the	ordinates $x = -2$ , $x = 3$ is equal to:
	(A) 2	(B)	
	(C) 4	(D)	8
	2 2		
79.	The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:		
	(A) 2πab	(B)	πab
	(C) $\frac{\pi ab}{2}$	(D)	None of these
	2	` /	

80. The area bounded by the curve  $y^2 = x$  and  $x^2 = y$  is given by :

(A) 0 (B) (C)  $\frac{2}{3}$  (D) 1

81. The whole area of the curve  $r = a \cos 2\theta$  is:

(A)  $\frac{\pi a^2}{2}$  (B)  $\pi a^2$  (C)  $2\pi a^2$  (D)  $\frac{2\pi a^2}{3}$ 

82. The line y = x + 1 is revolved about x-axis. The volume of solid of revolution formed by revolving the area covered by the given curve, x-axis and the lines x = 0, x = 2 is:

- (A)  $\frac{19\pi}{3}$  (B)  $\frac{17\pi}{3}$
- (C)  $\frac{13h}{3}$  (D)

83. The volume generated by revolution of the ellipse

about major axis is

[assume that a > b]:

(A)  $\frac{4\pi ab^2}{3}$ 

(B)  $\frac{4\pi a^2 b}{3}$ 

(C)  $\frac{4\pi a^2 b^2}{3}$ 

- (D) None of these
- 84. The surface of the solid of revolution about x-axis of the area bounded by the curve y = x, x-axis and the ordinates x = 0 and x = 3 is equal to:
  - (A)  $4\sqrt{2}\pi$

(B)  $9\sqrt{2}\pi$ 

(C)  $11\sqrt{2} \pi$ 

(D)  $8\sqrt{2} \pi$ 

- 85. The value of  $\int_{0}^{\frac{\pi}{2}} \sin^{6} x \, dx = :$ 
  - (A)  $\frac{5\pi}{8}$

(B)



 $\mathbf{C}$ 

(D)

- 86.
  - (A) 0

(B)  $\cos \pi^3$ 

(C)  $2\cos^3\pi$ 

- (D) Does not exist
- 87. Order and degree of the differential equation  $\sqrt{2\left(\frac{dy}{dx}\right)^3 + 4} = \left(\frac{d^2y}{dx^2}\right)^{3/2}$  are respectively:
  - (A) order 2, degree 3

(B) order 1, degree 3

(C) order 3, degree 2

(D) order 3, degree 1

- 88. If P, Q are functions of x, then solution of differential equation  $\frac{dy}{dx} + Py = Q$  is:
  - (A)  $ye^{\int Pdx} = \int Qe^{\int Pdx} dx + c$
- (B)  $y = e^{\int P dx} \int Q e^{\int P dx} dx + C$

(C)  $y = \int Q e^{\int P dx} dx + C$ 

- (D) None of these
- 89. The differential equation of the form  $\frac{dy}{dx} + Py = Qy^n$  where P and Q are functions of x, is called:
  - (A) Auxiliary equation

(B) Bessel's equation

(C) Clairaut's equation

- (D) Bernoulli's equation
- 90. The solution of  $(y \cos x + 1) dx + \sin x dy = 0$  is:
  - (A)  $x y \sin x = cx$

(B)  $y + x \sin x = c$ 

(C)  $y - x \sin x = c$ 

- (D)  $x + y \sin x = c$
- 91. If at every point of a certain curve the slope of the tangent equals  $\frac{-2x}{y}$ , the curve is:
  - (A) A straight line

(B) A parabola

(C) A circle

- (D) An ellipse
- 92. The integrating factor for the differential equation  $(x^2y 2xy^2) dx (x^3 3x^2y) dy$  is given by:
  - (A)

(B) xy

(C)  $x^2y^2$ 

- (D)  $\frac{1}{x^2y^2}$
- 93. The general solution of  $P = \log(px y)$  is:
  - (A)  $y = cx e^c$

(B)  $y + cx = e^{c}$ 

(C)  $y + x = \log c$ 

- (D)  $y + c = e^x$
- 94. The general solution of a differential equation of first order represents:
  - (A) A family of surfaces

- (B) A pair of curves in xy plane
- (C) A family of curves in xy plane
- (D) None of these

95. The singular solution of the differential equation 
$$P^3 + Px - y = 0$$
 is where  $P = \frac{dy}{dx}$ :

(A) 
$$27y^2 + 4x^3 = 0$$

(B) 
$$y^2 = 4ax$$

(C) 
$$x^2 + y^2 = a^2$$

(D) None of these

96. The orthogonal trajectory of the family of curves 
$$ay^2 = x^3$$
 is:

(A) 
$$3y^2 - 2x^2 = constant$$

(B) 
$$2x^2 + y^2 = constant$$

(C) 
$$3x^2 + y^2 = constant$$

(D) 
$$2x^2 + 3y^2 = constant$$

97. Solution of 
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$
 is:

(A) 
$$c_1 e^{-2x} + c_2 e^x$$

(B) 
$$c_1 e^{2x} + c_2 e^x$$

(A) 
$$c_1 e^{-2x} + c_2 e^x$$
  
(C)  $c_1 e^{2x} + c_2 e^{-2x}$ 

(B)  $c_1e^{2x} + c_2e^x$ (D) None of these

98. The general solution of the differential equation 
$$D^2(D+1)^2 y = e^x$$
 is:

(A) 
$$y = c_1 + c_2 x + (c_3 + c_4 x)e^x$$

(B) 
$$y = c_1 + c_2 x + (c_3 + c_4 x)e^{-x} + \frac{e^x}{4}$$

(C) 
$$y = c_1 + c_2 e^{-x} + (c_3 + c_4 x) e^{-x} + \frac{e^x}{4}$$

(D) None of these

99. The particular integral of the differential equation 
$$(D+2)(D-1)^3y = e^x$$
 is:

(A) 
$$\frac{x^3 e^x}{18}$$

(C) 
$$\frac{x^3 e^x}{3}$$

(D) None of these

100. The equation of the cylinder whose generators are parallel to the line 
$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$$
 and whose guiding curve is  $x^2 + 2y^2 = 1$ ,  $z = 0$  is given by :

(A) 
$$(3z-x)^2 + 2(2z + 3y)^2 = 9$$

(B) 
$$(3x+z)^2 + 2(3y-2z)^2 = 9$$

(C) 
$$(3x-z)^2 + 2(3y + 2z)^2 = 9$$

(D) 
$$(2z+3x)^2 + 2(3y-x)^2 = 9$$