

PHYSICS
Paper - I

Time Allowed : **Three Hours**

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Useful Constants :

Electron charge (e)	$= 1.602 \times 10^{-19} \text{ C}$
Electron rest mass (m_e)	$= 9.109 \times 10^{-31} \text{ kg}$
Proton mass (m_p)	$= 1.672 \times 10^{-27} \text{ kg}$
Vacuum permittivity (ϵ_0)	$= 8.854 \times 10^{-12} \text{ farad/m}$
Vacuum permeability (μ_0)	$= 1.257 \times 10^{-6} \text{ henry/m}$
Velocity of light in free space (c)	$= 3 \times 10^8 \text{ m/s}$
Boltzmann constant (k)	$= 1.380 \times 10^{-23} \text{ J/K}$
Electron volt (eV)	$= 1.602 \times 10^{-19} \text{ J}$
Planck constant (h)	$= 6.626 \times 10^{-34} \text{ Js}$
Stefan constant (σ)	$= 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$
Avogadro number (N)	$= 6.022 \times 10^{26} \text{ kmol}^{-1}$
Gas constant (R)	$= 8.31 \times 10^3 \text{ J kmol}^{-1} \text{ K}^{-1}$
exp (1)	$= 2.718$

SECTION A

Q1. Answer the following :

8×5=40

- (a) A bead slides on a wire in the shape of a cycloid described by the equations

$$x = a(\theta - \sin \theta)$$

$$y = a(1 + \cos \theta) \text{ with } 0 \leq \theta \leq 2\pi.$$

Find the Lagrangian and equation of motion.

8

- (b) Show that the relativistic invariance laws of conservation of momentum lead to the concepts of variation of mass with velocity and mass energy equivalence.

8

- (c) A parallel beam of light of wavelength 5890 \AA is incident at an angle of 30° on a plane transmission grating with 15000 lines/inch. Find the highest order of spectrum that can be observed.

8

- (d) Discuss absorption loss in an optical fibre comparing and contrasting the intrinsic and extrinsic absorption mechanisms.

8

- (e) Although the principle of operation of a basic LASER is based upon two energy levels, why does one need a 3-level or a 4-level scheme to achieve satisfactory lasing ? Explain your answer with special reference to a Ruby-laser.

8

Q2. (a) Discuss the mechanics of a system of point particles with special emphasis on the conservation theorems. How can we extend the results to a system with continuous mass distribution ?

10

- (b) (i) State and prove Hamilton's principle and use it to prove that the shortest distance between two points in space is a straight line joining them.

- (ii) Use Hamiltonian mechanics to find the differential equation for planetary motion, moving under force $f(r) = -\frac{k}{r^2}$ and prove that the areal velocity is constant.

8+7=15

- (c) (i) What is Holography ?

- (ii) Show with simple diagrams, how a hologram is written and read using a laser.

- (iii) Mention some important applications of holography.

3+8+4=15

- Q3.** (a) State the fundamental postulates of Einstein's special theory of relativity. Deduce Lorentz transformation equation and discuss how this accounts for the phenomenon of length contraction. 10
- (b) Discuss the properties of Cornu spiral. Show that the spiral can be used to obtain the intensity distribution in the Fresnel's diffraction pattern due to a straight edge. 10
- (c) (i) Using the concept of spontaneous and stimulated emission of radiation, obtain the relation between Einstein's A and B coefficients.
- (ii) What is the physical significance of Einstein's A coefficient ?
- (iii) Justify why lasing action is much more difficult at X-ray frequency than in case of infrared frequency spectrum. 10+5+5=20
- Q4.** (a) (i) In a Michelson's interferometer, 100 fringes cross the field of view when the movable mirror is displaced through 2.894×10^{-3} cm. Calculate the wavelength of the monochromatic source of light.
- (ii) A shift of 200 fringes is observed when the movable mirror of a Fabry-Pérot interferometer is shifted by 0.0298 mm. Calculate the wavelength of the incident radiation. 8+7=15
- (b) State and explain Fermat's principle of extremum path and use the same to deduce the laws of reflection and refraction of light. 10
- (c) (i) Explain the reason for pulse broadening due to intermodal and material dispersion. Deduce the relation of pulse broadening for intermodal dispersion in optical fiber.
- (ii) A step index fiber in air has a numerical aperture of 0.16, a core refractive index of 1.45 and a core diameter of 60 μm . Determine the normalized frequency for the fiber when light at a wavelength of 0.8 μm is transmitted. Also estimate the number of guided modes propagating in the fiber. 10+5=15

SECTION B

Q

Q5. Answer the following :

8×5=40

- (a) In a one-dimensional device, the charge density is given by

$$\rho_V = \rho_0 \frac{x}{a}.$$

If $E = 0$ at $x = 0$ and $V = 0$ at $x = a$,

find V and E using Laplace equation of electrostatics. 8

- (b) State and explain the Biot-Savart law. Derive an expression for the magnetic field at a point due to an infinitely long straight current carrying conductor. 8
- (c) Write down the four Maxwell's equations and explain the contribution of Maxwell in the development of these equations. 8
- (d) Prove the thermodynamic relation :

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

and hence show that

$$\frac{dP}{dt} = \frac{L}{T(V_2 - V_1)};$$

all the terms have their usual meanings. 8

- (e) Describe neutron star on the basis of Fermi-Dirac statistics and obtain the condition of critical mass for a neutron star. 8

G

- Q6.** (a) Use the method of electric images to find the electric field on the surface of a grounded conducting sphere. 10
- (b) (i) State Faraday's law of electromagnetic induction and prove that it can be expressed in the following vector form :

$$\text{Curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

with \vec{E} and \vec{B} being the electric and magnetic fields.

- (ii) A coil of 10 turns has dimension $9 \text{ cm} \times 7 \text{ cm}$. It rotates at the rate of $15\pi \text{ rad/sec}$ in a uniform field whose flux density is 0.6 weber/m^2 . What is the maximum e.m.f. induced in the coil? 10+5=15

- (c) How does one explain the observed spectrum of black-body radiation using Planck's quantum hypothesis ? State and obtain Wien's displacement law. Also explain the important features of this law. 15

- Q7.** (a) (i) Using Maxwell's equations, obtain the relation

$$\frac{1}{c} \frac{\partial}{\partial t} \left(\frac{E^2 + B^2}{2} \right) + \nabla \cdot (\bar{E} \times \bar{B}) = 0$$

- (ii) What is Poynting vector ? Deduce Poynting theorem for the flow of energy in an electromagnetic field. 5+10=15
- (b) Discuss the reflection and refraction of plane electromagnetic waves at plane dielectric boundaries for normal incidence and also find the reflection and transmission coefficients. 15
- (c) What do you understand by spontaneous magnetization below Curie temperature ? Explain with an appropriate diagram, the occurrence of a hysteresis loop in a ferromagnetic material. 10

- Q8.** (a) State Maxwell's distribution law of molecular speeds. Draw and explain a curve between $n(c)$ and c in a gas at a given temperature T , where $n(c) dc$ is the number of molecules having speed between c and $c + dc$. Discuss the effect of T and mass m of the molecule on the nature of the curve. 15
- (b) (i) Define and explain the significance of the quality factor of an electrical machine.
- (ii) Discuss in brief, the working principle of a transformer. 5+5=10
- (c) Derive the mathematical expression for the total energy of a degenerate Fermi gas at a temperature T and calculate the specific heat of the Fermi gas at this temperature. 15

