(1) Consider $A = \{ q \in \mathbb{Q} \, : \, q^2 \geq 2 \}$ as a subset of the metric space $(\mathbb{Q}, d)$, where $d(x, y) = |x - y|$. Then A is

A) closed but not open in $\mathbb{Q}$
B) open but not closed in $\mathbb{Q}$
C) neither open nor closed in $\mathbb{Q}$
D) both open and closed in $\mathbb{Q}$.

(2) The set $\mathbb{N}$ considered as a subspace of $(\mathbb{R}, d)$ where $d(x, y) = |x - y|$, is

A) closed but not complete
B) complete but not closed
C) both closed and complete
D) neither closed nor complete.

(3) Let Y be a totally bounded subset of a metric space X. Then the closure $\overline{Y}$ of Y

A) is totally bounded
B) may not be totally bounded even if X is complete
C) is totally bounded if and only if X is complete
D) is totally bounded if and only if X is compact.

(4) Let X, Y be metric spaces, $f : X \to Y$ be a continuous function, A be a bounded subset of X and let $B = f(A)$. Then B is

A) bounded
B) bounded if A is also closed
C) bounded if A is compact
D) bounded if A is complete.

(5) Let X be a connected metric space and U be an open subset of X. Then

A) U cannot be closed in X
B) if U is closed in X, then $U = X$
C) if U is closed in X, then $U = \phi$, the empty set
D) if U is closed in X and U is non-empty, then $U = X$.

(6) Let X be a connected metric space and $f : X \to \mathbb{R}$ be a continuous function. Then $f(X)$

A) is whole of $\mathbb{R}$
B) is a bounded subset of $\mathbb{R}$
C) is an interval in $\mathbb{R}$
D) may not be an interval in $\mathbb{R}$.

(7) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,\, y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Let $D_u f(0,\, 0)$ denote the directional derivative of f at $(0,\, 0)$ in the direction $u = (u_1,\, u_2) \neq (0,\, 0)$. Then f is

A) continuous at $(0, 0)$ and $D_u f(0,\, 0)$ exist for all u
B) continuous at $(0, 0)$ but $D_u f(0,\, 0)$ does not exist for some $u \neq (0,\, 0)$
C) not continuous at $(0, 0)$ but $D_u f(0,\, 0)$ exist for all u
D) not continuous at $(0, 0)$ and $D_u f(0,\, 0)$ does not exist for some $u \neq (0,\, 0)$.

(8) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,\, y) = \frac{x^2 - y^2}{1 + x^2 + y^2}$$

Then

A) $\frac{\partial^2 f}{\partial x \partial y}(0,\, 0)$ and $\frac{\partial^2 f}{\partial y \partial x}(0,\, 0)$ exist but are not equal
B) $\frac{\partial^2 f}{\partial x \partial y}(0,\, 0)$ exist but $\frac{\partial^2 f}{\partial y \partial x}(0,\, 0)$ does not exist
C) $\frac{\partial^2 f}{\partial y \partial x}(0,\, 0)$ exist but $\frac{\partial^2 f}{\partial x \partial y}(0,\, 0)$ does not exist
D) $\frac{\partial^2 f}{\partial x \partial y}(0,\, 0)$ and $\frac{\partial^2 f}{\partial y \partial x}(0,\, 0)$ exist and are equal.

(9) The sequence

$$\left\langle \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n} \right\rangle$$

converges to

A) 1 B) 2 C) 3 D) 5.

(10) The limit of the sequence $\langle \sqrt{(n+1)(n+2)} - n \rangle$ as $n \to \infty$ is

A) $\sqrt{2} - 1$ B) 3 C) $3/2$ D) 0.

(11) The radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{1}{2^n} x^{3n}$$

is

A) 1 B) ∞ C) $1/2$ D) $2^{1/3}$.

(12) Which one of the following sequence converges uniformly on the indicated set?

A) $f_n(x) = (1 - |x|)^n; \quad x \in (-1,\, 1)$

B) $f_n(x) = \frac{1}{n}\sin nx; \quad x \in \mathbb{R}$
C) $f_n(x) = x^n; \quad x \in [0,\, 1]$
D) $f_n(x) = \frac{1}{1+x^n}; \quad x \in [0, \infty)$.

(13) Which one of the following integrals is convergent?

A) $\int_1^\infty \frac{1}{x^2}dx$ B) $\int_1^\infty \frac{1}{\sqrt{x}}dx$
C) $\int_0^1 \frac{1}{x^2}dx$ D) $\int_0^\infty \frac{1}{\sqrt{x}}$.

(14) The value of the integral

$$\int_{-\infty}^{\infty} e^{-x^2}dx$$

is

A) 0 B) $\sqrt{2\pi}$ C) $\sqrt{\pi}$ D) $\sqrt{\pi/2}$.

(15) Let $f : I \to \mathbb{R}$ be an increasing function where I is an interval in $\mathbb{R}$. Then

A) f^2 is always increasing
B) f^2 is always decreasing
C) f^2 is constant $\Rightarrow f$ is constant
D) f^2 may be neither decreasing nor increasing.

(16) Consider the function $f(x) = x^2$ on $[0,\, 1]$ and the partition P of $[0,\, 1]$ given by

$$P = \left\{ 0 < \frac{1}{n} < \frac{2}{n} < \cdots < \frac{n-1}{n} < 1 \right\}.$$

Then the upper and the lower Riemann sums of f are

A) $U(f,\, P) = (1+\frac{1}{n})(2-\frac{1}{n})/6$ and $L(f,\, P) = (1+\frac{1}{n})(2+\frac{1}{n})/6$
B) $U(f,\, P) = (1+\frac{1}{n})(2+\frac{1}{n})/6$ and $L(f,\, P) = (1-\frac{1}{n})(2-\frac{1}{n})/6$
C) $U(f,\, P) = (1+\frac{1}{n})(2+\frac{1}{n})/6$ and $L(f,\, P) = (1-\frac{1}{n})(2+\frac{1}{n})/6$
D) $U(f,\, P) = (1-\frac{1}{n})(2+\frac{1}{n})/6$ and $L(f,\, P) = (1+\frac{1}{n})(2-\frac{1}{n})/6$.

(17) Which one of the following is true?

A) If $\sum a_n$ diverges and $a_n > 0$, then $\sum \frac{a_n}{1+a_n}$ diverges
B) If $\sum a_n$ and $\sum b_n$ diverge, then $\sum(a_n + b_n)$ diverges
C) If $\sum a_n$ and $\sum b_n$ diverge, then $\sum(a_n + b_n)$ converges
D) If $\sum a_n$ converges and $\sum b_n$ diverges, then $\sum(a_n + b_n)$ converges.

(18) If $\sum a_n = A$, $\sum |a_n| = B$ and A and B are finite, then

A) $|A| = B$ B) $A \leq B$
C) $|A| \geq B$ D) $A = B$.

(19) If $x_n = 1 + (-1)^n + \frac{1}{2^n}$, then

A) $\limsup x_n = 1$
B) $\liminf x_n = 1$
C) x_n is a convergent sequence
D) $\limsup x_n \neq \liminf x_n$.

(20) Let $\langle x_n \rangle$ be the sequence defined by $x_1 = 2$ and $x_{n+1} = \frac{1}{2}(x_n + \frac{2}{x_n})$. Then

A) $\langle x_n \rangle$ converges to rational number
B) $\langle x_n \rangle$ is an increasing sequence
C) $\langle x_n \rangle$ converges to $2\sqrt{2}$
D) $\langle x_n \rangle$ is a decreasing sequence.

(21) Which one of the following series converges?

A) $\sum \cos \frac{1}{n^2}$
B) $\sum \sin \frac{1}{n^2}$
C) $\sum \frac{1}{n^{1+1/n}}$
D) $\sum n^{\cos 3}$.

(22) The sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}$$

is

A) $\frac{\pi^2}{8}$
B) $\frac{\pi^2}{6}$
C) $\frac{\pi}{2}$
D) 1.

(23) Which one of the following set is not countable?

A) $\mathbb{N}^r$, where $r \geq 1$ and $\mathbb{N}$ is the set of natural numbers
B) $\{0, 1\}^{\mathbb{N}}$, the set of all the sequences which takes values 0 and 1
C) $\mathbb{Z}$, set of integers
D) $\sqrt{2}\mathbb{Q}$, $\mathbb{Q}$ is set of rational numbers.

(24) Let $f : [0, 1] \to \mathbb{R}$ be a continuous function such that $f(x^2) = f(x)$ for all $x \in [0, 1]$. Which one of the following is not true in general?

A) f is constant
B) f is uniformly continuous
C) f is differentiable
D) $f(x) \geq 0 \; \forall \, x \in [0, 1]$.

(25) Let $f : [0, 1] \to [0, 1]$ be a continuous function and $I : [0, 1] \to [0, 1]$ be the identity function. Then f and I

A) agree exactly at one point
B) agree at least at one point
C) may not agree at any point
D) agree at most at one point.

(26) For $x \in \mathbb{R}$, let $[x]$ denote the greatest integer n such that $n \leq x$. The function $h(x) = x[x]$ is

A) continuous everywhere
B) continuous only at $x = \pm 1, \pm 2, \pm 3, \cdots$
C) continuous if $x \neq \pm 1, \pm 2, \pm 3, \cdots$
D) bounded on $\mathbb{R}$.

(27) Let $\langle x_n \rangle$ be an unbounded sequence in $\mathbb{R}$. Then

A) $\langle x_n \rangle$ has a convergent subsequence
B) $\langle x_n \rangle$ has a subsequence $\langle x_{n_k} \rangle$ such that $x_{n_k} \to 0$
C) $\langle x_n \rangle$has a subsequence $\langle x_{n_k} \rangle$ such that $\frac{1}{x_{n_k}} \to 0$
D) Every subsequence of $\langle x_n \rangle$ is unbounded.

(28) Consider the function $g : \mathbb{R} \to \mathbb{R}$ defined by

$$g(x) = \begin{cases} 0, & \text{if } x \geq 0, \\ e^{-1/x^2}, & \text{if } x < 0. \end{cases}$$

Which one of the following is not true?

A) g has derivatives of all orders at every point
B) $g^n(0) = 0$ for all $n \in \mathbb{N}$
C) Taylor Series expansion of g about $x = 0$ converges to g for all x
D) Taylor Series expansion of g about $x = 0$ converges to g for all $x \geq 0$.

(29) The function

$$f(x) = x \sin x + \frac{1}{1+x^2}; \quad x \in I$$

where $I \subseteq \mathbb{R}$ is

A) uniformly continuous if $I = \mathbb{R}$
B) uniformly continuous if I is compact
C) uniformly continuous if I is closed
D) not uniformly continuous on $[0, 1]$.

(30) Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x^2, & \text{if } x \in (0,\,2) \cap \mathbb{Q}, \\ 2x-1, & \text{if } x \in (0,\,2) \cap (\mathbb{R} \setminus \mathbb{Q}). \end{cases}$$

Which one of the following is not true?

A) f is continuous at $x = 1$
B) f is differentiable at $x = 1$
C) f is not differentiable at $x = 1$
D) f is differentiable only at $x = 1$.

(31) Let R be a finite commutative ring with unity and P be an ideal in R satisfying: $ab \in P \implies a \in P$ or $b \in P$, for any $a, b \in R$. Consider the statements:
(i) P is a finite ideal
(ii) P is a prime ideal
(iii) P is a maximal ideal.
Then

A) (i),(ii) and (iii) are all correct
B) None of (i),(ii) or (iii) is correct
C) (i) and (ii) are correct but (iii) is not correct
D) (i) and (ii) are not correct but (iii) is correct.

(32) Let $\phi : R \to R'$ be a non-zero mapping such that $\phi(a+b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$ for all $a, b \in R$, where R, R' are rings with unity. Then

A) $\phi(1) = 1$ for all rings with unity R, R'
B) $\phi(1) \neq 1$ for any rings with unity R, R'
C) $\phi(1) \neq 1$ if R' is an integral domain or if ϕ is onto
D) $\phi(1) = 1$ if R' is an integral domain or if ϕ is onto.

(33) Let R be a ring, L be a left ideal of R and let $\lambda(L) = \{x \in R \mid xa = 0\ \forall a \in L\}$. Then

A) $\lambda(L)$ is not a two-sided ideal of R
B) $\lambda(L)$ is a two-sided ideal of R
C) $\lambda(L)$ is a left but not right ideal of R
D) $\lambda(L)$ is a right but not left ideal of R.

(34) Let $S = \{a + ib \mid a, b \in \mathbb{Z}, b \text{ is even}\}$. Then

A) S is both a subring and an ideal of $\mathbb{Z}[i]$
B) S is neither an ideal nor a subring of $\mathbb{Z}[i]$
C) S is an ideal of $\mathbb{Z}[i]$ but not a subring of $\mathbb{Z}[i]$
D) S is a subring of $\mathbb{Z}[i]$ but not an ideal of $\mathbb{Z}[i]$.

(35) The set of all ring homomorphism $\phi : \mathbb{Z} \to \mathbb{Z}$

A) is an infinite set
B) has exactly two elements
C) is a singleton set
D) is an empty set.

(36) Let F be a field of characteristic 2. Then

A) either F has 2^n elements or is an infinite field
B) F is an infinite field
C) F is a finite field with 2^n elements
D) either F is an infinite field or a finite field with $2n$ elements.

(37) Consider the following classes of commutative rings with unity: ED is the class of Euclidean domain, PID is the class of principal ideal domain, UFD is the class of unique factorization domain and ID is the class of integral domain. Then

A) PID $\subset$ ED $\subset$ UFD $\subset$ ID
B) ED $\subset$ UFD $\subset$ PID $\subset$ ID
C) ED $\subset$ PID $\subset$ UFD $\subset$ ID
D) UFD $\subset$ PID $\subset$ ED $\subset$ ID.

(38) Consider the polynomial ring $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$. Then

A) $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$ both are Euclidean domains
B) $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$ both are not Euclidean domains
C) $\mathbb{Z}[x]$ is a Euclidean domain but $\mathbb{Q}[x]$ is not a Euclidean domain
D) $\mathbb{Q}[x]$ is a Euclidean domain but $\mathbb{Z}[x]$ is not a Euclidean domain.

(39) Let R be a commutative ring with unity such that the polynomial ring $R[x]$ is a principal ideal domain. Then

A) R is a field
B) R is a PID but not a field
C) R is a UFD but not a field
D) R is not a field but is an integral domain.

(40) Let T be a linear transformation on $\mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3)$. What is T^{-1}?

A) $T^{-1}(x_1, x_2, x_3) = (\frac{x_1}{3}, \frac{x_1}{3} + x_2, -x_1 + x_2 + x_3)$
B) $T^{-1}(x_1, x_2, x_3) = (\frac{x_1}{3}, \frac{x_1}{3} - x_2, x_1 + x_2 + x_3)$
C) $T^{-1}(x_1, x_2, x_3) = (\frac{x_1}{3}, \frac{x_1}{3} - x_2, -x_1 + x_2 + x_3)$
D) $T^{-1}(x_1, x_2, x_3) = (\frac{x_1}{3}, \frac{x_1}{3} + x_2, x_1 + x_2 + x_3)$.

(41) Let V be the vector space of all $n \times n$ matrices over a field F. Which one of the following is not a subspace of V?

A) All upper triangular matrices of order n
B) All non-singular matrices of order n
C) All symmetric matrices of order n
D) All matrices of order n, the sum of whose diagonal entries is zero.

(42) Let V be the vector space of all $n \times n$ matrices over a field. Let V_1 be the subspace of V consisting of all symmetric matrices of order n and V_2 be the subspace of V consisting of all skew-symmetric matrices of order n. Which one of the following is not a subspace of V?

A) $V_1 + V_2$ B) $V_1 \cup V_2$ C) $V_1 \oplus V_2$ D) $V_1 \cap V_2$.

(43) Let $V = \mathbb{R}^3$ be the real inner product space with the usual inner product. A basis for the subspace $u^\perp$ of V, where $u = (1, 3, -4)$, is

A) $\{(1,0,3),(0,1,4)\}$, B) $\{(3,-1,0),(-6,2,0)\}$
C) $\{(-3,1,0),(4,0,1)\}$ D) $\{(3,1,0),(-4,0,1)\}$.

(44) The matrix A that represents the linear operator T on $\mathbb{R}^2$, where T is the reflection in $\mathbb{R}^2$ about the line $y = -x$ is

A) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
B) $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
C) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
D) $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

(45) Consider the subspace U of $\mathbb{R}^4$ spanned by the vectors $v_1 = (1,1,1,1), v_2 = (1,1,2,4), v_3 = (1,2,-4,-3)$. An orthonormal basis of U is

A) $\{\frac{1}{2}(1,1,1,1), \frac{1}{\sqrt{6}}(-1,-1,0,2), \frac{1}{\sqrt{2}}(1,3,-6,2)\}$
B) $\{\frac{1}{2}(1,1,1,1), \frac{1}{2\sqrt{6}}(-1,-1,0,2), \frac{1}{\sqrt{2}}(1,3,6,-2)\}$
C) $\{\frac{1}{2}(1,1,1,1), \frac{1}{\sqrt{6}}(-1,-1,0,2), \frac{1}{5\sqrt{2}}(1,3,-6,2)\}$
D) $\{(1,1,1,1),(-1,-1,0,2),(1,3,-6,2)\}$.

(46) Let V be a vector space over $\mathbb{Z}_5$ of dimension 3. The number of elements in V is

A) 5 B) 125 C) 243 D) 3.

(47) Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors $u_1 = (1, -2, 5, -3), u_2 = (2, 3, 1, -4), u_3 = (3, 8, -3, -5)$. The dimension of W is

A) 1 B) 2 C) 3 D) 4.

(48) Let λ be a non-zero characteristic root of a non-singular matrix A of order 2×2. Then a characteristic root of the matrix adj.A is

A) $\frac{\lambda}{|A|}$ B) $\frac{|A|}{\lambda}$ C) $\lambda|A|$ D) $\frac{1}{\lambda}$.

(49) Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ be a 2×2 matrix. Then the expression $A^5 - 2A^4 - 3A^3 + A^2$ is equal to

A) $2A + 3I$ B) $3A + 2I$ C) $2A - 3I$ D) $3A - 2I$.

(50) The number of elements in the group $Aut\ \mathbb{Z}_{200}$ of all automorphisms of $\mathbb{Z}_{200}$ is

A) 78 B) 80 C) 84 D) 82.

(51) Let $A = \begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$ be a matrix over the integers modulo 11. The inverse of A is

A) $A = \begin{pmatrix} 8 & 9 \\ 10 & 9 \end{pmatrix}$
B) $A = \begin{pmatrix} 10 & 8 \\ 9 & 9 \end{pmatrix}$
C) $A = \begin{pmatrix} 9 & 10 \\ 9 & 8 \end{pmatrix}$
D) $A = \begin{pmatrix} 9 & 9 \\ 10 & 8 \end{pmatrix}$.

(52) The order of the group $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| ad - bc = 1 \text{ and } a, b, c, d \in \mathbb{Z}_3 \right\}$ relative to matrix multiplication is

A) 18 B) 20 C) 24 D) 22.

(53) The number of subgroups of the group $\mathbb{Z}_{200}$ is

A) 8 B) 14
C) 12 D) 10.

(54) Let $G = U(32)$ and $H = \{1, 31\}$. The quotient group G/H is isomorphic to

A) $\mathbb{Z}_8$
B) $\mathbb{Z}_4 \oplus \mathbb{Z}_2$
C) $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$
D) The dihedral group D_4.

(55) The number of sylow 5-subgroups of the group $\mathbb{Z}_6 \oplus \mathbb{Z}_5$ is

A) 6 B) 4 C) 12 D) 1.

(56) The singular solution of the first order differential equation $p^3 - 4xyp + 8y^2 = 0$ is

A) $27x - 4y^3 = 0$ B) $27y - 4x^2 = 0$
C) $27y - 4x^3 = 0$ D) $27y + 4x^3 = 0$.

(57) The general solution of the system of first order differential equations

$$\frac{dx}{dt} + \frac{dy}{dt} = x + t,$$
$$\frac{dx}{dt} - \frac{d^2y}{dt^2} = 0$$

is given by

A) $x = \frac{1}{2}t + c_1t^2 + c_2t;\ y = \frac{1}{2}t - c_1t + c_2$
B) $x = \frac{1}{2}t^2 + c_1t + c_2;\ y = \frac{1}{6}t^3 + \frac{1}{2}c_1t^2 + (c_2 - c_1)t + c_3$
C) $x = \frac{1}{2}t^2 - c_1t + c_2t^2;\ y = \frac{1}{6}t^2 + \frac{1}{2}c_1t^2 + (c_2 - c_1)t^2 + c_3$
D) $x = \frac{1}{3}t^2 + c_1t + c_2;\ y = \frac{1}{6}t^3 - \frac{1}{2}c_1t + (c_2 - c_1)t^2 + c_3$.

(58) Consider the following statements regarding the two solutions $y_1(x) = \sin x$ and $y_2(x) = \cos x$ of $y'' + y = 0$:
(i) They are linearly dependent solutions of $y'' + y = 0$.
(ii) Their wronskian is 1.
(iii) They are linearly independent solutions of $y'' + y = 0$.
which of the statements is true?

A) (i) and (ii) B) (ii) and (iii)
C) (iii) D) (i).

(59) The general solution of $\frac{d^4y}{dx^4} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$ is

A) $y = c_1 + c_2x + c_3x^2 + c_4e^x$
B) $y = c_1 - c_2x + c_3x^3 + c_4e^{-x}$
C) $y = (c_1 + c_2x + c_3x^2)e^{2x} + c_4e^x$

D) $y = (c_1 + c_2x + c_3x^2)e^{2x} + c_4e^{-x}$.

(60) The solution of the initial value problem $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0,\ y(0) = -3,\ y'(0) = -1$ is

A) $y = e^{3x}(2\cos 4x + 3\sin 2x)$
B) $y = e^{-3x}(2\sin 2x - 3\cos 2x)$
C) $y = e^{3x}(2\sin 4x - 3\cos 4x)$
D) $y = e^{3x}(2\sin 4x + 3\cos 4x)$.

(61) The sturm-Liouville problem given by $y'' + \lambda y = 0,\ y(0) = 0,\ y(\pi) = 0$ has a trivial solution if

A) $\lambda \leq 0$ B) $\lambda > 0$
C) $0 < \lambda < 1$ D) $\lambda \geq 1$.

(62) The initial value problem $y' = 1 + y^2, y(0) = 1$ has the solution given by

A) $y = \tan(x - \frac{\pi}{4})$
B) $y = \tan(x + \frac{\pi}{4})$
C) $y = \tan(x - \frac{\pi}{2})$
D) $y = \tan(x + \frac{\pi}{2})$.

(63) The series expansion that gives y as a function of x in neighborhood of $x = 0$ when $\frac{dy}{dx} = x^2 + y^2$; with boundary conditions $y(0) = 0$ is given by

A) $y = \frac{1}{3}x^3 + \frac{1}{63}x^7 + \frac{2}{2079}x^{11} + \cdots$
B) $y = \frac{1}{2}x^3 + \frac{1}{8}x^5 + \frac{1}{32}x^7 + \cdots$
C) $y = x^2 + \frac{1}{2!}x^3 + \frac{1}{3!}x^4 + \cdots$
D) $y = \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \cdots$.

(64) The value of $y(0.2)$ obtained by solving the equation $\frac{dy}{dx} = \log(x + y),\ y(0) = 1$ by modified Euler's method is equal to

A) 1.223 B) 1.0082
C) 2.381 D) 1.639.

(65) Reciprocal square root iteration formula for $N^{-1/2}$ is given by

A) $x_{i+1} = \frac{x_i}{2}(3 - x_i^2N)$
B) $x_{i+1} = \frac{x_i}{9}(4 - x_i^2N)$
C) $x_{i+1} = \frac{1}{16}(8 - x_i^2N)$
D) $x_{i+1} = \frac{x_i}{4}(10 - x_i^2N)$.

(66) If the formula $\int_0^h f(x)\, dx = h[af(0) + bf(\frac{h}{3}) + cf(h)]$ is exact for polynomials of as high order as possible, then $[a, b, c]$ is

A) $[0, 2, 3]$ B) $[1, 5, \frac{9}{4}]$
C) $[\frac{3}{4}, 2, 9]$ D) $[0, \frac{3}{4}, \frac{1}{4}]$.

(67) If f is continuous, $f(x_1)$ and $f(x_2)$ are of opposite sign and $f(\frac{x_1+x_2}{2})$ has same sign as $f(x_1)$, then

A) $(\frac{x_1+x_2}{2}, x_2)$ must contain at least one zero of $f(x)$
B) $(\frac{x_1+x_2}{2}, x_2)$ contain no zero of $f(x)$
C) $(x_1, \frac{x_1+x_2}{2})$ must contain at least one zero of $f(x)$
D) $(\frac{x_1+x_2}{2}, x_2)$ has no zero of $f(x)$.

(68) The first iteration solution of system of equations

$$\begin{aligned} 2x_1 - x_2 &= 7 \\ -x_1 + 2x_2 - x_3 &= 1 \\ -x_2 + 2x_3 &= 1 \end{aligned}$$

by Gauss-Seidel method with initial approximation $x^{(0)} = 0$ is

A) $[3.5, 2.25, 1.625]$
B) $[4.625, 3.625, 2.315]$
C) $[5, 3, 1]$
D) $[5.312, 4.312, 2.656]$.

(69) The partial differential equation for the family of surfaces $z = ce^{\omega t}\cos(\omega x)$, where c and ω are arbitrary constants, is

A) $z_{xx} + z_{tt} = 0$
B) $z_{xx} - z_{tt} = 0$
C) $z_{xt} + z_{tt} = 0$
D) $z_{xt} + z_{xx} = 0$.

(70) The integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x - y = 0$, $z = 1$ is

A) $x^2 + y^2 + 2xyz - 2z + 2 = 0$
B) $x^2 + y^2 - 2xyz - 2z + 2 = 0$
C) $x^2 + y^2 - 2xyz + 2z + 2 = 0$
D) $x^2 + y^2 + 2xyz + 2z + 2 = 0$.

(71) The solution of heat equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ for which a solution tends to zero as $t \to \infty$ is

A) $z(x,t) = \sum_{n=0}^{\infty} c_n \cos(nx + \epsilon_n) e^{-n^2kt}$
B) $z(x,t) = \sum_{n=0}^{\infty} c_n \cos(nx + \epsilon_n) e^{n^2kt}$
C) $z(x,t) = \sum_{n=0}^{\infty} c_n \sin(nx + \epsilon_n) e^{n^2kt}$
D) $z(x,t) = \sum_{n=-\infty}^{\infty} c_n \sin(nx + \epsilon_n) e^{n^2kt}$.

(72) The complete integral of the equation $p^2y(1 + x^2) = qx^2$ is

A) $z = a(1 + x^2) + \frac{1}{2}a^2y^2 + b$
B) $z = \frac{1}{2}a^2\sqrt{1 + x^2} + a^2y^2 + b$
C) $z = a\sqrt{1 + x^2} + \frac{1}{2}a^2y^2 + b$
D) $z = a(1 + x^2) + \frac{1}{2}ay + b$.

(73) The general integral of the partial differential equation $z(xp - yq) = y^2 - x^2$ is

A) $x^2 + y^2 + z^2 = f(xy)$
B) $x^2 - y^2 + z^2 = f(xy)$
C) $x^2 - y^2 - z^2 = f(xy)$
D) $x^2 + y^2 - z^2 = f(xy)$.

(74) The solution of the partial differential equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$ is

A) $z = x\phi_1(x + y) + \phi_2(x + y) + x\psi_1(x + y) + \psi_2(x + y)$
B) $z = x\phi_1(x - y) + \phi_2(x - y) + x\psi_1(x - y) + \psi_2(x - y)$
C) $z = x\phi_1(x + y) + \phi_2(x - y) + x\psi_1(x + y) + \psi_2(x - y)$
D) $z = x\phi_1(x - y) + \phi_2(x - y) + x\psi_1(x + y) + \psi_2(x + y)$.

(75) The eigen values and eigen functions of the vibrating string problem $u_{tt} - c^2u_{xx} = 0$, $0 \le x \le l$, $t > 0$, $u(x, 0) = f(x)$, $0 \le x \le l$, $u_t(x, 0) = g(x)$, $0 \le x \le l$, $u(0, t) = 0$, $u(l, t) = 0$, $t \ge 0$ are

A) $(\frac{n\pi}{l})^2, \sin\frac{n\pi x}{l}, n = 1, 2, 3, \cdots$
B) $(\frac{n\pi}{l})^2, \cos\frac{n\pi x}{l}, n = 1, 2, 3, \cdots$
C) $\frac{n\pi}{l}, \sin\frac{n\pi x}{l}, \cos\frac{n\pi x}{l}, n = 1, 2, 3, \cdots$
D) All the above.