

GGSIPO Mathematics 2004

1. If the angles between the pair of straight lines represented by the equation

$$x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0 \text{ is } \tan^{-1} \frac{1}{3}.$$

Where ' λ ' is a non-negative real number, then λ is :

- a 2 b 0
c 3 d 1

2. The distance of the line $2x - 3y = 4$ from the point $(1, 1)$ measured parallel to the line $x + y = 1$ is :

- a $\sqrt{2}$ b $5/\sqrt{2}$
c $1/\sqrt{2}$ d 6

3. The equations of bisectors of the angles between the lines $|x| = |y|$ are :

- a $y = \pm x$ and $x = 0$
b $x = \frac{1}{2}$ and $y = \frac{1}{2}$
c $y = 0$ and $x = 0$
d none of these

4. The base of vertices of an isosceles triangle PQR are Q $(1, 3)$ and R $(-2, 7)$. The vertex P can be :

- a $(1, 6)$ b $(\frac{1}{2}, 5)$
c $(\frac{5}{6}, 6)$ d none of these

5. The normal at the point $(3, 4)$ on a circle cuts the circle at the point $(-1, -2)$. Then the equation of the circle is :

- a $x^2 + y^2 + 2x - 2y - 13 = 0$
b $x^2 + y^2 - 2x - 2y - 11 = 0$
c $x^2 + y^2 - 2x + 2y + 12 = 0$
d $x^2 + y^2 - 2x - 2y + 14 = 0$

6. If $\cos P = \frac{1}{7}$ and $\cos Q = \frac{13}{14}$ where 'P' and 'Q' both are acute angles. Then the value of P-Q is :

- a 30° b 60°

c 45° d 75°

7. The equation $3 \cos x + 4 \sin x = 6$ has solution

a finite b infinite

c one d no

8. If $\sec^{-1} x = \operatorname{cosec}^{-1} y$, then $\cos^{-1} \frac{1}{x} + \cos^{-1} \frac{1}{y}$ is equal to :

a π b $\pi/4$

c $-\pi/2$ d $\pi/2$

9. If 'n' be any integer, then $n^{n+1} 2^{n+1}$ is :

a odd number b integral

multiple of 6

c perfect square d does not

necessarily have any of the foregoing proof

10. If $\tan \theta = -\frac{4}{3}$, then the value of $\sin \theta$ is :

a $-\frac{4}{5}$ but $\neq \frac{4}{5}$ b $-\frac{4}{5}$ or $\frac{4}{5}$

c $\frac{4}{5}$ but $\neq -\frac{4}{5}$ d $\frac{1}{5}$

11. If $c = 2 \cos \theta$, then the value of the determinant $\begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix}$ is :

a $\frac{\sin 4 \theta}{\sin \theta}$ b $\frac{2 \sin^2 2\theta}{\sin \theta}$

c $4 \cos^2 \theta - 2 \cos \theta - 1$ d none of these

12. the set of values of x for which the inequality $|x-1| + |x+1| < 4$ always holds true is :

a $-2, 2$ b $-\infty, 2 \cup$

$2, \infty$

c $-\infty, 1] \cup [1, \infty$ d none

of these.

13. The equation of the parabola whose vertex is $-1,-2$, axis is vertical and which passes through the point $3,6$, is :

a $x^2 + 2x - 2y - 3 = 0$

b $2x^2 = 3y$

c $x^2 - 2x + 2y - 3 = 0$

d $x^2 - 2x - 2y - 3 = 0$

14. The length of the axis of the conic $9x^2 + 4y^2 - 6x + 4y + 1 = 0$ are :

a $\frac{1}{2}, 9$ b $3, \frac{2}{5}$

(c) $\frac{2}{3}$ d $3, 2$

15. If $f(x) = \cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ and $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{g(x)-g(a)}$, $0 < a < \frac{1}{2}$, is :

a $\frac{3}{2(1+a^2)}$ b $\frac{3}{2(1+x^2)}$

c $\frac{3}{2}$ d $-\frac{3}{2}$

16. If $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2x - 1, & 1 < x \leq 1 \end{cases}$ then :

= 1

= 1

not differentiable at $x = 1$

a f is discontinuous at x

b f is differentiable at x

c f is continuous but

d none of these

17. $\lim_{x \rightarrow -2} \frac{\sin^{-1}(x+2)}{x^2 + 2x}$ is equal to :

$x = -2$

a 0 b ∞

c $-\frac{1}{2}$ d none of these

18. Let $f(x) = x^p \cos \frac{1}{x}$, when $x \neq 0$ and $f(x) = 0$, when $x = 0$. then $f(x)$ will be differentiable at $x = 0$, if :

- a $p > 0$ b $p > 1$
 c $0 < p < 1$ d $\frac{1}{2} < p < 1$

1

19. The derivative of $f(x) = 3|2+x|$ at the point $x_0 = -3$ is :

- a 3 b -3
 c 0 d none

of these

20. Derivative of the function $f(x) = \log_5(\log_7 x)$, $x > 7$ is :

- a $\frac{1}{x \log_5(\log_7)(\log_7 x)}$
 b $\frac{1}{x(\log_5)(\log_7)}$
 c $\frac{1}{x \log x}$
 d none of these

21. If $z = x+iy$, $z^{1/3} = a - ib$, then $\frac{x}{a} - \frac{y}{b} = k a^2 - b^2$, where k is equal to :

- a 1 b 2
 c 3 d 4

22. The number of real solutions of the equation $1 + |e^x - 1| = e^x e^{-x} - 2$ is :

- a 1 b 2
 c 4 d 8

23. The points of extrema of $f(x) = \int_0^x \frac{\sin t}{t} dt$ in a domain $x > 0$ are :

a $2n+1 \frac{\pi}{2}$, $n =$

1,2,.....

b $4n+1 \frac{\pi}{2}$, $n =$

1,2,.....

1,2,.....

c $2n+1 \frac{\pi}{4}, n =$

d $n \pi, n = 1, 2, \dots$

24. If $x^2 + y^2 = t^2$ and $x=s+3t, y=2s-t$, then $\frac{d^2u}{ds^2}$ is equal to :

a 12 b 10

c 32 d 36

25. If the equation $x^2+qx+p = 0$ have a common root then $p+q+1$ is equal to :

a 0 b 1

c 2 d -1

26. The value of a and b for which the sum of the cubes of the roots of $x^2 - a - 2x + a - 3 = 0$ assumes the last value is :

a 3 b 4

c 5 d

none of these

27. Let z_1, z_2, z_3 be three vertices of an equilateral triangle circumscribing the circle $|z| = \frac{1}{2}$. If $z_1 = \frac{1}{2} + \frac{i\sqrt{3}}{2}$ and z_1, z_2, z_3 were in anticlockwise sense, then z_2 is :

1- $\overline{3i}$

a $1 + \overline{3i}$ b

c 1 d -1

28. If $z = \frac{-2}{1 + \sqrt{3}i}$, then the value of $\arg z$ is :

$\pi/3$

a π b

$\pi/4$

c $2\pi/3$ d

29. Let ω is an imaginary cube roots of unity, then the value of

$21 + \omega + \omega^2 + 32 + \omega + 12 + \omega^2 + 1 + \dots + n + 1 + n + \omega + 1 + n + \omega^2 + 1$ is :

- a $\left[\frac{n(n+1)}{2}\right]^2 + n \left[\frac{n^2(n+1)^2}{4}\right]$
 c $\left[\frac{n(n+1)}{2}\right]^2 - n$ d none of these

30. The locus of the point z satisfying $\arg\left(\frac{z-1}{z+1}\right) = k$, (where k is non zero) is :

- a a circle with centre on y-axis
 b circle with centre on x-axis
 c a straight line parallel to x-axis
 d a straight line making an angle 60° with the x-axis

31. If $P(3,4,5), Q(4,6,3), R(-1,2,4), S(1,0,5)$, then the projection of RS on PQ is :

- a $-\frac{2}{3}$ b $-\frac{4}{3}$
 c $\frac{1}{2}$ d 2

32. If a line makes α, β, γ with the positive direction of x, y, z-axes respectively. Then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ is equal to :

- a $\frac{1}{2}$ b $-\frac{1}{2}$
 c -1 d 1

33. The projection of a line on co-ordinate axes are 2,3,6. Then the length of the line is :

- a 7 b 5
 c 1 d 11

34. The decimal equivalent of the binary number 10011.1 is :

- a 19.50 b 11001.11
 c 5005.55
 d 19.10

35. The binary represents of 60 is :

111100

a 101110 b

110000

c 110011 d

36. Which of the following statement is not tautology ?

b $p \vee q \wedge p$

a $\sim p \vee q \vee p$

d $\sim p \vee q \wedge \sim p \vee p$

c $q \vee \sim p \vee q$

37. The period of $f(x) = \sin\left(\frac{rx}{n-1}\right) + \cos\left(\frac{rx}{n}\right)$, $n \in \mathbb{Z}, n > 2$ is :

b $4\pi n - 1$

a $2\pi n - 1$

d none of these

c $2\pi n - 1$

39. The radius of the circle whose arc of length 15 km makes an angle of $\frac{3}{4}$ radian at the centre, is :

b 20 cm

a 10 cm

d $22 \frac{1}{2}$ cm

c $11 \frac{1}{4}$ cm

40. If $f_n(x) = e^{f_{n-1}(x)}$, for all $n \in \mathbb{N}$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is equal to :

b $f_n(x) \cdot \frac{d}{dx}\{f_{n+1}(x)\}$

a $f_n(x) \cdot f_{n-1}(x)$

.... $f_2(x) \cdot f_1(x)$ d none of these

c $f_n(x) \cdot f_{n-1}(x)$

41. if $3^x + 2^{2x} \geq 5^x$, then the solution set for x is :

b $[2, \infty)$

a $[-\infty, 2]$

d {2}

c [0,2]

42. The number of integral solution of $\frac{x+1}{x^2+2} > \frac{1}{4}$ is :

b 2

a 1

d none of these

c 5

43. The value of k for which the equation $k - 2x^2 + 8x + k + 4 = 0$ has both real, distinct and -ve, is :

b 2

a 0

d -4

c 3

44. The triangle PQR of which the angles P, Q, R satisfy $\cos P = \frac{\sin Q}{2 \sin R}$:

b right angled

a equilateral

d isosceles

c any triangle

45. If $f(x) = a - x^{n-1/n}$, where $a > 0$ and n is a positive integer, then $f[f(x)]$ is equal to :

b x^{-2}

a x^{-3}

d none of these

c x

46. The function $f(x) = [x]^2 - [x^2]$ where $[y]$ is the greatest integer less than or equal to y is discontinuous at :

except 0 and 1

a all integers

b all integers

except 0

c all integers

except 1

d all integers

47. the function $f(x) = |px - q| + r|x|$, $x \in -\infty, \infty$ where $p > 0, q, r > 0$ assumes its maximum value only at one point, if :

$q \neq r$

a $p \neq q$ b

$p = q = r$

c $r \neq p$ d

48. A function $f(x) = \frac{x^2 - 3x + 2}{x^2 + 2x - 3}$ is :

$x = -3$

a maximum at

at $x = -3$ and maximum at $x = 1$

b maximum

$x = 1$

c maximum at

increasing in its domain

d function is

49. The locus of the point (x, y) satisfying the relation

$$\sqrt{(x-3)^2 + (y-1)^2} + \sqrt{(x+3)^2 + (y-1)^2} = 6 \text{ is}$$

- a Straight line
- b Pair of straight lines
- c Circle
- d Ellipse

50. If z_1, z_2 and z_3 are complex number such that $|z_1| = |z_2| = |z_3| = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 1$ then $|z_1 + z_2 + z_3|$ is :

a equal to 1

b less than 1

than 3

c greater

d equal to 3

51. Let a_1, a_2, a_3 be any positive real numbers, then which of the following statement is not true ?

$a_1^3 + a_2^3 + a_3^3$

a $3a_1 a_2 a_3 \leq$

$\frac{a_3}{a_1} \geq 3$

b $\frac{a_1}{a_2} + \frac{a_2}{a_3} +$

$\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \geq 9$

c $a_1 a_2 a_3$

$\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^3 \leq 27$

d $a_1 a_2 a_3$

52. If $ab = 2a + 3b, a > 0, b < 0$, then the minimum value of ab is :

b 24

a 12

d none of these

c $\frac{1}{4}$

53. Let N be +ve integer $\neq 1$, then none of the numbers $2, 3, \dots, N$ is divisor of $N! - 1$. So we can conclude that $N! - 1$ is :

number

a prime

one of this number $N+1, N+2, \dots, N! - 2$ is divisor of $N! - 1$

b at least

smallest numbers between N and $N!$ which is divisor of $N! - 1$ is prime number

c The

these

d none of

54. If $f(x) = \cos[\pi^2 x] + \cos[-\pi^2 x]$, then :

a $f(\pi/4) = 2$

- b $f^{-\pi}=2$
- c $f^{\pi}=1$
- d $f^{\pi/2}=-1$

55. let $fx = \frac{x^2-4}{x^2+4}$, for $|x| > 2$, then the function $f : -\infty, -2] \cup [2, \infty \rightarrow -1, 1$ is :

- a one - one into
- b one - one onto
- c many - one into
- d many-one onto

56. The function $f(x = \sin \log x + \sqrt{x^2 + 1})$ is :

- a even function
- b odd function
- c even nor odd
- d periodic function

57. The range of $f(x = \sec(\frac{\pi}{4} \cos^2 x))$, $-\infty < x < \infty$ is :

- a $[1, \sqrt{2}]$
- b $[1, \infty)$
- c $[-\sqrt{2}, -1] \cup [1, \infty)$
- d $[-\infty, 1] \cup [1, \infty)$

58. For any three sets A_1, A_2, A_3 . Let $B_1 = A_1, B_2 = A_2 - A_1$ and $B_3 = A_3 - A_1 \cup A_2$, then which of the following statement is always true ?

- a $A_2 \cup A_3 \supset B_1 \cup B_2 \cup B_3$
- b $A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$
- c $A_2 \cup A_3 \subset B_1 \cup B_2 \cup B_3$
- d none of these

59. the domain of the function $f(x = \frac{\sin^{-1}(3-x)}{\log(x-2)})$ is :

b 3,4]

a [2,4]

d $-\infty, 3 \cup [2, \infty$

c [2, ∞

60. The remainder obtained when $1! + 2! + \dots + 200!$ is divided by 14 is :

b 4

a 3

d none of these

c 5