

GGSIPIU mathematics 2008

1. Let \vec{u} and \vec{v} be two equal vectors inclined at an angle θ , then a $\sin \frac{\theta}{2}$ is equal to

- a $\frac{|\vec{u}-\vec{v}|}{2}$ b $\frac{|\vec{u}+\vec{v}|}{2}$
 c $|\vec{u}-\vec{v}|$ d $|\vec{u}+\vec{v}|$

2. $\int \frac{dx}{x^2+4x+13}$ is equal to

- a $\log(x^2+4x+13) + c$
 b $\frac{1}{3} \tan^{-1} \left(\frac{x+2}{3} \right) + c$
 c $\log(2x^2+4) + c$
 d $\frac{2x+4}{(x^2+4x+13)^2} + c$

3. The general solution $y^2 dx + x^2 - xy + y^2 dy = 0$ is

- a $\tan^{-1} \left(\frac{x}{y} \right) \cdot \log y + c = 0$
 b $2 \tan^{-1} \left(\frac{x}{y} \right) + \log x + c = 0$
 c $\log y + \sqrt{x^2 + y^2} + \log y + c = 0$
 (d) $\sinh^{-1} \left(\frac{x}{y} \right) + \log y + c = 0$

4. $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x) dx$ is equal to

- a $\sqrt{2} - 2$ b $2\sqrt{2} - 2$
 c $3\sqrt{2} - 2$ d $4\sqrt{2} - 2$

5. Out of 40 consecutive natural numbers, two are chosen at random. Probability that the sum of the number is odd, is

- a $\frac{14}{29}$ b $\frac{20}{39}$
 c $\frac{1}{2}$ d None of these

6. Equation of tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, which are perpendicular to the line $3x+4y = 7$, are

a $4x - 3y = \pm \sqrt{20}$ b $4x - 3y = \pm \sqrt{12}$

c $4x - 3y = \pm \sqrt{1}$ d $4x - 3y = \pm 1$

7. If \vec{a} is perpendicular to \vec{b} and \vec{c} , $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 4$ and the angle between \vec{b} and \vec{c} is $\frac{2\pi}{3}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to

a $4\sqrt{3}$ b $6\sqrt{3}$

c $12\sqrt{3}$ d $18\sqrt{3}$

8. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is

a $y = \frac{1}{4}e^{-2x} + \frac{cx}{2} + d$

b $y = \frac{1}{4}e^{-2x} + cx + d$

c $y = \frac{1}{4}e^{-2x} + cx^2 + d$

d) $y = \frac{1}{4}e^{-2x} + cx^3 + d$

9. The value of $\int_2^3 \frac{x+1}{x^2(x-1)} dx$ is

a $\log \frac{16}{9} + \frac{1}{6}$ b $\log \frac{16}{9} - \frac{1}{6}$

c $2 \log 2 - \frac{1}{6}$ d $\log \frac{4}{3} - \frac{1}{6}$

10. The length of the chord of the parabola $x^2 = 4y$ passing through the vertex and having slope $\cot \alpha$ is

a $4 \cos \alpha \operatorname{cosec}^2 \alpha$ b $4 \tan \alpha \sec \alpha$

c $4 \sin \alpha \sec^2 \alpha$ d None of these

11. The records of a hospital show that 10% of the cases of a certain disease, then the probability that only three will die, is

a 8748×10^{-5} b 1458×10^{-5}

c 1458×10^{-6} d 41×10^{-6}

12. From the point P(16,7) tangents PQ and PR are drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. If c be the centre of the circle, then area of quadrilateral PQCR is

a 450 sq unit b 15 sq unit

- c 50 sq unit d 75 sq unit

13. If $\tan x = \frac{b}{a}$, then the value of $a \cos 2x + b \sin 2x$ is

- a a b a - b
c a+b d b

14. In a triangle ABC, right angled at C, the value of $\cot A + \cot B$ is

- a $\frac{c^2}{ab}$ b $a+b$
c $\frac{a^2}{bc}$ d $\frac{b^2}{ac}$

16. If α, β are roots of the equation $lx^2 + mx + n = 0$, then the equation whose roots are $\alpha^3 \beta$ and $\alpha \beta^3$, is

- a $lx^2 - nlm^2 - 2nlx + n^4 = 0$
b $lx^2 + nlm^2 - 2nlx + n^4 = 0$
c $lx^2 + nlm^2 - 2nlx - n^4 = 0$
d $lx^2 - nlm^2 - 2nlx + n^4 = 0$

17. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32} \dots$

- a 3/2 b 5/2
c 2 d 1

18. $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \end{bmatrix}$ is equal to

- a $\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$ b $\begin{bmatrix} 2 & -1 & -1 \\ -2 & -1 & 1 \\ 4 & 2 & -2 \end{bmatrix}$
(c) [-1] d not defined

19. $\lim_{x \rightarrow \infty} \frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}$ is equal to

- a $\frac{1}{10}$ b 00
c $\frac{1}{5}$ (c) $\frac{3}{10}$

20. Function $f(x) = \begin{cases} x-1, & x < 2 \\ 2x-3, & x \geq 2 \end{cases}$ is a continuous function

- a for $x = 2$ only
- b for all real values of x such that $x \neq 2$
- c for all real values of x
- d for all integral values of x only

21. Differential coefficient of $\sqrt{\sec \sqrt{x}}$ is

- a $\frac{1}{4\sqrt{x}} \sec \sqrt{x} \sin \sqrt{x}$
- b $\frac{1}{4\sqrt{x}} (\sec \sqrt{x})^{3/2} \cdot \sin \sqrt{x}$
- c $\frac{1}{2} \sqrt{x} \sec \sqrt{x} \sin \sqrt{x}$
- d $\frac{1}{2} \sqrt{x} \sec \sqrt{x}^{3/2} \cdot \sin \sqrt{x}$

22. The function $x^5 - 5x^4 + 5x - 1$ is

- a neither maximum nor minimum at $x = 0$
- b maximum at $x = 0$
- c maximum at $x = 1$ and minimum at $x = 3$
- (d) minimum at $x = 0$

23. If $x = y \sqrt{1 - y^2}$, then $\frac{dy}{dx}$ is equal to

- a $\frac{\sqrt{1 - y^2}}{1 + 2y^2}$
- b $\frac{\sqrt{1 - y^2}}{1 - 2y^2}$
- c $\frac{\sqrt{1 - y^2}}{1 - 2y^2}$
- d 0

24. If the planes $x + 2y + kz = 0$ and $2x + y - 2z = 0$, are at the right angles, then the value of k is

- a 1
- b -2
- c $\frac{1}{2}$
- d $-\frac{1}{2}$

25. The ratio in which the line joining $(2, 4, 5)$, $(3, 5, -4)$ is divided by the yz plane is

- a 2:3
- b 3:2
- c -2:3
- d 4:-3

26. If the lines $3x+4y+1=0$, $5x+\lambda y+3=0$ and $2x+y-1=0$ are concurrent, then λ is equal to

- a -8 (b) 8 c 4 d -4

27. The value of $\int_0^1 \frac{x^4+1}{x^2+1} dx$ is

- a $\frac{1}{6} 3 - 4\pi$ b $\frac{1}{6} 3 \pi + 4$
 c $\frac{1}{6} 3 + 4 \pi$ d $\frac{1}{6} 3 \pi - 4$

28. The solution of the differential equation

$$\frac{dy}{dx} = y \tan x - 2 \sin x, \text{ is}$$

- a $Y \sin x = c + \sin 2x$
 b $Y \cos x = c + \frac{1}{2} \sin 2x$
 c $Y \cos x = c - \sin 2x$
 d $Y \cos x = c + \frac{1}{2} \cos 2x$

29. The value of $1 - \log 2 + \frac{(\log 2)^2}{2!} - \frac{(\log 2)^3}{3!} + \dots$ is

- a $\log 3$ b $\log 2$
 c $\frac{1}{2}$ d None of these

30. The maximum value of $f(x) = \frac{x}{4+x+x^2}$ on $[-1,1]$ is

- a $\frac{1}{3}$ b $-\frac{1}{4}$
 c $\frac{1}{5}$ d $\frac{1}{6}$

31. If the radius of a circle be increasing at a uniform rate of 2 cm/s. The area of increasing of area of circle, at the instant when the radius is 20 cm, is

- a $70 \pi \text{ cm}^2 / \text{s}$ b $70 \text{ cm}^2 / \text{s}$
 c $80 \pi \text{ cm}^2 / \text{s}$ d $80 \text{ cm}^2 / \text{s}$

32. If $PA = PB = x$ and $\angle AOB = \angle BPA' = \angle B' = \frac{1}{3}$, then x is equal to

- a $\frac{1}{2}$ b $\frac{1}{3}$
 c $\frac{1}{4}$ d $\frac{1}{6}$

33. The focus of the parabola $y^2 - x - 2y + 2 = 0$ is

a $(\frac{1}{4}, 0)$ b $(1, \frac{1}{4})$

c $(\frac{5}{4}, 1)$ d $(\frac{3}{4}, \frac{5}{2})$

34. The equation of normal at the point 0,3 of the ellipse $9x^2 + 5y^2 = 45$ is

a x -axis b y -axis

c $y+3=0$ d $y -3=0$

35. The equation of the tangent parallel to $y-x+5=0$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

a $x -y+1=0$ b $x -y+2=0$

c $x+y-1=0$ d $x+y-2=0$

36. Let the functions f,g,h are defined from the set of real numbers R to R such that $f(x) = x^2 - 1$, $g(x) = \sqrt{x^2 - 1}$ and $hx = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

Then $h \circ f \circ g(x)$ is defined by

a X

b X^2

c 0

d None of these

37. The angle of elevation of the sun, if the length of the shadow of a tower is $\sqrt{3}$ times the height of the pole, is

a 150° b 30°

c 60° d 45°

38. If $\sin A = n \sin B$, then $\frac{n-1}{n+1} \tan \frac{A+B}{2}$ is equal to

a $\sin \frac{A-B}{2}$ b $\tan \frac{A-B}{2}$

c $\cot \frac{A-B}{2}$ d None of these

39. $3 \tan^{-1} a$ is equal to

a $\tan^{-1} \frac{3a+a^3}{1+3a^2}$

b $\tan^{-1} \frac{3a-a^3}{1+3a^2}$

c $\tan^{-1} \frac{3a+a^3}{1-3a^2}$

d $\tan^{-1} \frac{3a-a^3}{1-3a^2}$

40. In which quadrant of the complex plane, the point $\frac{1+2i}{1-i}$ lies ?

a Fourth b First

c Second d Third

41. If $\sin \alpha$ and $\cos \alpha$ are the roots of the equation $px^2+qx+r = 0$, then

a $p^2+q^2-2pr = 0$

b $p^2-q^2+2pr = 0$

c $p^2-q^2-2pr = 0$

d $p^2+q^2+2pr = 0$

42. If a, b, c are in the GP, then the equations $ax^2+2bx+c = 0$ and $dx^2+2ex+f = 0$ have a common root, if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

a AP b GP

c HP d None of these

43. In the expansion of $2x^2 - \frac{1}{x}^{12}$, the term independent of x is

a 8th b 7th

c 9th d 10th

44. The general value of θ in the equation $\cos \theta = \frac{1}{\sqrt{2}}, \tan \theta = -1$ is

a $2n\pi \pm \frac{\pi}{6}, n \in \mathbb{I}$

b $2n\pi + \frac{7\pi}{6}, n \in \mathbb{I}$

c $n\pi + (-1)^n \frac{\pi}{3}, n \in \mathbb{I}$

d $n\pi + (-1)^n \frac{\pi}{4}, n \in \mathbb{I}$

45. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$, then A^{-1} is equal to

a $\begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}$

b $\begin{bmatrix} 5/11 & 2/11 \\ 3/11 & -1/11 \end{bmatrix}$

c $\begin{bmatrix} -5/11 & -2/11 \\ -3/11 & -1/11 \end{bmatrix}$

d $\begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix}$

46. The value of $\lim_{x \rightarrow \infty} \left(\frac{x^2 + bx + 4}{x^2 + ax + 5} \right)$ is

a $\frac{b}{a}$ b 0

c 1 (d) $\frac{4}{5}$

47. Let $f(x) = \begin{cases} \frac{\sin \pi x}{5x} & x \neq 0 \\ k & x = 0 \end{cases}$ if $f(x)$ is continuous at $x = 0$, then k is equal to

a $\frac{\pi}{5}$ b $\frac{5}{\pi}$

c 1 d 0

48. If θ be the angle between the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$, then

a $\cos \theta = \frac{4}{21}$

b $\cos \theta = \frac{3}{19}$

c $\cos \theta = \frac{2}{19}$

d $\cos \theta = \frac{5}{21}$

49. Let \vec{a}, \vec{b} and \vec{c} be vectors with magnitudes 3, 4 and 5 respectively and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the values of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is

a 47 b 25

c 50 d -25

50. The maximum value of $z = 4x + 2y$ subjected the constrains $2x + 3y \leq 18, x + y \leq 10, x, y \geq 0$

- a 20 b 36**
c 40 d None of these