

ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION

**MATHEMATICS: PAPER - I**

Time: 3 hours

Full marks 100

*(Group - A is compulsory. Attempt any FOUR questions from Group - B).*

**GROUP - A**

**(This Group is COMPULSORY)**

1: Attempt any ten (10) questions:

2 x 10=20

(i) The function  $f(x) = |x - 1|$  is

- (a) continuous at  $x = 1$
- (b) not derivable at  $x = 1$
- (c) both (a) and (b) are true
- (d) none of these

(ii) The value of the integral  $\int \frac{e^t}{1 + e^t} dt =$

- (a)  $1 + e^t$
- (b)  $\log(1 + e^t)$
- (c)  $[\log(1 + e^t)]^{-1}$
- (d)  $[1 + e^t]^{-1}$

(iii) The set  $\mathbf{N}$  of natural numbers is

- (a) bounded
- (b) bounded below
- (c) bounded above
- (d) not bounded

(iv) Every convergent sequence is

- (a) bounded
- (b) Cauchy
- (c) both (a) and (b) true
- (d) none of these

(v) A subgroup  $H$  of a group  $G$  is normal if and only if

- (a)  $xH \subset Hx^{-1}, \forall x \in G$
- (b)  $xH \subseteq Hx, \forall x \in G$
- (c)  $xHx^{-1} \supset H, \forall x \in G$
- (d)  $xHx^{-1} = H, \forall x \in G$

(vi) The set  $A = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \dots\}$  is a

- (a) closed subset of  $R$ ,
- (b) open subset of  $R$ ,
- (c) both closed and open subset of  $R$ ,
- (d) neither closed nor open subset of  $R$ .

(vii) Let  $G = \{-1, 1\}$  be a group under multiplication. Define  $f : Z \rightarrow G$  by

$$f(n) = \begin{cases} 1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd} \end{cases}, \text{ where } (Z, +) \text{ is the group of integers. Then}$$

- (a)  $f$  is an isomorphism
- (b)  $f$  is an endomorphism
- (c)  $f$  is an automorphism
- (d)  $f$  is an epimorphism

(viii) The set  $S = \{a + ib, c + id\}$  is a basis of the space  $C(R)$  if and only if

- (a)  $ad - bc \neq 0$
- (b)  $ab - cd \neq 0$
- (c)  $ac - bd \neq 0$
- (d)  $ad - bc = 0$

(ix) Consider the mapping  $T : R^2 \rightarrow R^2$  such that  $T(1, 0) = (1, 1)$  and  $T(0, 1) = (-1, 2)$ . Then

- (a)  $T(x, y) = (x + y, x - 2y)$
- (b)  $T(x, y) = (x + y, x + 2y)$
- (c)  $T(x, y) = (x - y, x - 2y)$
- (d)  $T(x, y) = (x - y, x + 2y)$

(x) The matrix  $A = \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix}$  is invertible if

- (a)  $\alpha = \pm 1$
- (b)  $\alpha = \pm i$
- (c)  $\alpha \neq \pm 1$
- (d) none of these

(xi) The eigen values of the matrix  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$  are

- (a) 1, 6
- (b) 2, 3
- (c) 3, 4
- (d) 4, 6

(xii) The fuzzy set operations satisfy

- (a) Law of contradiction,
- (b) Law of excluded middle,
- (c) Both law of contradiction and law of excluded middle
- (d) None of the above property.

(xiii) The following linear programming problem:

Maximize  $z = x + y$  subject to the constraints:

$x + y \leq 1, -3x + y \geq 3, x \geq 0, y \geq 0$  has

- (a) a unique optimal solution,
- (b) infinite number of optimal solution
- (c) no feasible solution
- (d) unbounded solution.

(xiv) The bilinear transformation which maps the points  $z = \infty, i, 0$  into the points  $w = 0, i, \infty$ , respectively is

(a)  $w = \frac{1}{z}$

(b)  $w = -\frac{1}{z}$

(c)  $w = \frac{2}{z}$

(d)  $w = -\frac{2}{z}$

### **GROUP - B**

**(Answer any Four.)**

**2:** Attempt any two (2) questions:

(a) (i) Find the value of  $\lim_{x \rightarrow 0} \left( \frac{1 - \cos x}{x^2} \right)$ . (5)

(ii) Show that  $\int_0^{\infty} \frac{\sin x}{x^p} dx$  converges absolutely if  $p > 1$ . (5)

(b) Let  $f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$  (10)

Show that  $f_{xy} \neq f_{yx}$  at  $(0, 0)$ . Also find  $f_{xx}, f_{yy}$  at  $(1, 1)$ .

(c) Prove that  $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \beta(m, n)$ . (10)

**3:** Attempt any two (2) questions:

(a) (i) Let  $R$  be a ring and let  $a \in R$ . Show that the subset  $X = \{r \in R : ar = 0\}$  is a left ideal of  $R$ . (2)

(ii) Prove that every finite group  $G$  is isomorphic to a permutation group. (8)

(b) (i) Let  $Q^+$  be the set of positive rational numbers. Define an operation  $*$  on  $Q^+$  as  $a * b = \frac{ab}{3}$  for  $a, b \in Q^+$ . Verify that  $(Q^+, *)$  is an abelian group. (5)

(ii) Prove that every field is an integral domain. (5)

(c) (i) If every element of a group  $G$  is its own inverse, then show that  $G$  is abelian. (5)

(ii) Prove that the intersection of two ideals is an ideal. (5)

**4:** Attempt any two (2) questions:

(a) (i) Show that the three vectors  $(1, 1, -1)$ ,  $(2, -3, 5)$  and  $(-2, 1, 4)$  of  $R^3$  are linearly independent. (5)

(ii) Define rank of a matrix. Find the rank of the matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ . (5)

OR

(iii) Show that the union of two subspaces is a subspace if and only if one is contained in the other. (5)

(b) If  $U, W$  are two subspaces of a finite dimensional vector space  $V(F)$ , then prove that  $\dim(U+W) = \dim U + \dim W - \dim(U \cap W)$ . (10)

(c) Find the characteristic roots of the matrix  $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$  and verify Cayley-Hamilton theorem for the matrix. Also find  $A^{-1}$ . (3+4+3=10)

5: Attempt any two (2) questions:

(a) (i) Show that the function  $e^x(\cos y + i \sin y)$  is analytic. (5)

(ii) If a function  $f(z)$  is continuous on a contour  $C$  of length  $L$  and if  $M$  is the upper

bound of  $f(z)$  on  $C$ , prove that  $\left| \int_C f(z) dz \right| \leq ML$ . (5)

(b) State and prove Cauchy's Residue theorem. (10)

(c) State Cauchy's integral formula. Evaluate:  $\int_C \frac{\sin^8 z}{(z - \frac{\pi}{6})^4} dz$  if  $C$  is a circle  $|z|=1$ . (2+8=10)

6: Attempt any two (2) questions:

10 x 2=20

(a) Solve the following LPP:

Maximize  $z = 6x + 4y$  subject to the constraints:

$$2x + 3y \leq 30, 3x + 2y \leq 24, x + y \geq 3, x \geq 0, y \geq 0.$$

(b) Let  $(X, d)$  and  $(Y, \rho)$  be metric spaces. Prove that a function  $f: X \rightarrow Y$  is continuous at  $x_0 \in X$  iff  $x_n \rightarrow x_0$  in  $X$  implies  $f(x_n) \rightarrow f(x_0)$  in  $Y$ .

(c) Prove that a continuous image of a connected set is connected. Also, show that the property Hausdorff space is a hereditary property.

(d) Suppose a random variable  $X$  takes on the values  $-3, -1, 2$ , and  $5$  with respective probabilities

$$\frac{2k-3}{10}, \frac{k-2}{10}, \frac{k-1}{10}, \frac{k+1}{10}$$

(i) Determine the distribution of  $X$ ;

(ii) Find the expected value  $E(X)$  of  $X$ .

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