ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION

STATISTICS : PAPER - I

Time- 3 (Three) Hours   Full Marks- 100

Question No 1 is compulsory and answer any four from the rest.

1) Briefly describe or define any 10 (ten) of the following.   2 x 10 = 20.

(i) Bernoulli and Binomial random variable.

(ii) Bayes theorem

(iii) Exponential family of distributions

(iv) Minimum variance unbiased estimator

(v) Likelihood ratio test

(vi) Non central chi square distribution

(vii) Tchebyshev's inequality

(viii) Sufficiency

(ix) Completeness

(x) Circular systematic sampling

(xi) Probability proportional to size

(xii) Canonical and Standard LPP.

(xiii) Compact set

(xiv) Basis

(xv) G inverse.

2) Answer any 4 (four) of the following.   5 x 4 = 20

(i) State any prove Cayley Hamilton theorem.

(ii) Prove that if the n x n matrix A satisfies the equation $A^2 = A$, then Rank(A) + Rank (I-A) = n.

(iii) Explain the axiomatic approach to probability.

(iv) State and prove the memoryless property of exponential distribution. Explain its importance.

(v) State and prove the condition under which Poisson distribution tends to normal distribution.

(vi) Show that the distribution function follows continuous uniform distribution.

3) Answer any 4 (four) of the following. 5 x 4 = 20

(i) Discuss two different types of plots which are used to test violation of assumptions of linear regression model.

(ii) Explain why the Ordinary Least Square procedure cannot be used for regressing binary dependent variables.

(iii) Show that dual of the dual LPP is the primal LPP.

(iv) Compare the variance under linear systematic sampling and stratified random sampling technique.

(v) State and prove the Weak Law of Large numbers.

(vi) Let $X \sim N_2(0,\Sigma)$. Prove that $X' \Sigma^{-1} X$ follows the chi square distribution with 2 degrees of freedom.

4) Answer any 2 (two) of the following. 10 x 2 = 20

(i) What are orthogonal polynomials? Show how they are useful.

(ii) Explain any three different methods of estimation along with their properties.

(iii) State and prove the Lehmann Scheffe theorem and explain its usefulness.

5) Answer any 2 (two) of the following. 10 x 2 = 20

(i) Prove that in a multivariate normal distribution, any subset of the variables is distributed independently of the complementary subset if and only if every variable in the former set is uncorrelated with every variable in the later set.

(ii) Discuss the Weibull distribution along with its characteristics.

(iii) Discuss different characteristics of the Transportation problem along with a method of solution.

6) Answer any 2 (two) of the following. 10 x 2 = 20

(i) State and prove the Gauss Markov theorem and explain its importance in the study of ANOVA.

(ii) Define Measure, Probability Measure and Measurable functions with examples.

(iii) Let $g(y)$ be a continuous function and let $\{X_n\}$ be a sequence of random variables such that $X_n \rightarrow X$ in probability. Show that $g(X_n) \rightarrow g(X)$ in probability.

7) Derive the distribution of the sample correlation coefficient when population correlation coefficient is $\neq 0$.

$20 \times 1 = 20$

8) Suppose that $X_1, X_2, \ldots, X_n$ are n eigen vectors corresponding to the eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ of a matrix A of order n. Attempt all of the following:

$20 \times 1 = 20$

(i) If A is a symmetric matrix, then show that the eigen values are all real

(ii) Show that the eigen vectors $X_1, X_2, \ldots, X_n$ are linearly independent.

(iii) Since λ_1 is an eigen value of A, show that $1/\lambda_1$ is an eigen value of A^{-1}.

(iv) Show that Trace (A) = $\sum \lambda_i$.

9) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu,\sigma_1^2)$ and let $Y_1, Y_2, \ldots, Y_m$ be independently drawn sample from $N(\mu,\sigma_2^2)$. Find the m.l.e's of μ,σ_1^2 and σ_2^2. Also find the variance of these estimators.

$20 \times 1 = 20$

10) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(-\theta,\theta)$, $\theta>0$. Find the method of moment estimator of θ and show that it is a biased estimator.

$20 \times 1 = 20$

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