MATHEMATICS

1. If $A=\{a,b,c\}$, $B=\{b,c,d\}$ and $C=\{a,d,c\}$, then $(A-B)\times(B\cap C)=$

1) $\{(a,c),(a,d),(b,d)\}$
2) $\{(c,a),(d,a)\}$
3) $\{(a,b),(c,d)\}$
4) $\{(a,c),(a,d)\}$

2. The function $f:X\rightarrow Y$ defined by $f(x)=Sin\,x$ is one-one but not onto if X and Y are respectively equal to

1) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ and $[-1,1]$
2) $\left[0,\frac{\pi}{2}\right]$ and $[-1,1]$
3) $[0,\pi]$ and $[0,1]$
4) $\mathbb{R}$ and $\mathbb{R}$

3. If $Log_4^2+Log_4^4+Log_4^{16}+Log_4^x=6$, then $x=$

1) 32
2) 8
3) 4
4) 64

4. If $S_n=\frac{1}{6.11}+\frac{1}{11.16}+\frac{1}{16.21}+\ldots\ldots$ to n terms, then $6S_n=$

1) $\frac{1}{(5n+6)}$
2) $\frac{(2n-1)}{5n+6}$
3) $\frac{n}{(5n+6)}$
4) $\frac{5n-4}{5n+6}$

5. The remainder obtained when $(\underline{|1})^2+(\underline{|2})^2+(\underline{|3})^2+\ldots\ldots+(\underline{|100})^2$ is divided by 10^2 is

1) 14
2) 17
3) 28
4) 27

(Space for Rough Work)

6. If $(p \wedge \sim r) \rightarrow (\sim p \vee q)$ is false, then the truth values of p, q and r are respectively

1) T, F and T 2) F, T and T
3) F, F and T 4) T, F and F

7. If α, β and γ are the roots of the equation $x^3 - 8x + 8 = 0$, then $\sum \alpha^2$ and $\sum \frac{1}{\alpha\beta}$ are respectively =

1) 16 and 0 2) – 16 and 0
3) 16 and 8 4) 0 and – 16

8. The g.c.d. of 1080 and 675 is

1) 125 2) 225
3) 135 4) 145

9. If $a \mid (b+c)$ and $a \mid (b-c)$ where $a, b, c \in N$ then,

1) $c^2 \equiv a^2 \left(\text{mod } b^2\right)$ 2) $a^2 \equiv b^2 \left(\text{mod } c^2\right)$
3) $a^2 + c^2 = b^2$ 4) $b^2 \equiv c^2 \left(\text{mod } a^2\right)$

10. If a, b and $c \in N$ which one of the following is not true ?

1) $a \mid b$ and $a \mid c \Rightarrow a \mid b+c$ 2) $a \mid b+c \Rightarrow a \mid b$ and $a \mid c$
3) $a \mid b$ and $b \mid c \Rightarrow a \mid c$ 4) $a \mid b$ and $a \mid c \Rightarrow a \mid 3b+2c$

(Space for Rough Work)

11. If $2A+3B=\begin{bmatrix}2 & -1 & 4\\3 & 2 & 5\end{bmatrix}$ and $A+2B=\begin{bmatrix}5 & 0 & 3\\1 & 6 & 2\end{bmatrix}$, then $B=$

1) $\begin{bmatrix}8 & 1 & 2\\1 & 10 & 1\end{bmatrix}$
2) $\begin{bmatrix}8 & 1 & -2\\-1 & 10 & -1\end{bmatrix}$
3) $\begin{bmatrix}8 & 1 & 2\\-1 & 10 & -1\end{bmatrix}$
4) $\begin{bmatrix}8 & -1 & 2\\-1 & 10 & -1\end{bmatrix}$

12. If $O(A)=2\times3$, $O(B)=3\times2$, and $O(C)=3\times3$, which one of the following is not defined ?

1) $C(A+B')$
2) $C(A+B')'$
3) BAC
4) $CB+A'$

13. If $A=\begin{bmatrix}1 & -3\\2 & K\end{bmatrix}$ and $A^2-4A+10I=A$, then $K=$

1) 1 or 4
2) 4 and not 1
3) -4
4) 0

14. The value of $\begin{vmatrix}x+y & y+z & z+x\\x & y & z\\x-y & y-z & z-x\end{vmatrix}=$

1) 0
2) $(x+y+z)^3$
3) $2(x+y+z)^3$
4) $2(x+y+z)^2$

15. On the set Q of all rational numbers the operation * which is both associative and commutative is given by $a*b=$

1) $2a+3b$
2) $ab+1$
3) a^2+b^2
4) $a+b+ab$

(Space for Rough Work)

16. In the group $G = \{1,5,7,11\}$ under multiplication modulo 12, the solution of $7^{-1} \times_{12} \left(x \times_{12} 11 \right) = 5$ is $x =$

1) 11
2) 7
3) 1
4) 5

17. A subset of the additive group of real numbers which is not a sub group is

1) $(Q, +)$
2) $(N, +)$
3) $(Z, +)$
4) $(\{0\}, +)$

18. If $\vec{p} = \hat{i} + \hat{j}$, $\vec{q} = 4\hat{k} - \hat{j}$ and $\vec{r} = \hat{i} + \hat{k}$, then the unit vector in the direction of $3\vec{p} + \vec{q} - 2\vec{r}$ is

1) $\hat{i} + 2\hat{j} + 2\hat{k}$
2) $\frac{1}{3}\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$
3) $\frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right)$
4) $\frac{1}{3}\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

19. If $\vec{a}$ and $\vec{b}$ are the two vectors such that $\left|\vec{a}\right| = 3\sqrt{3}$, $\left|\vec{b}\right| = 4$ and $\left|\vec{a} + \vec{b}\right| = \sqrt{7}$, then the angle between $\vec{a}$ and $\vec{b}$ is

1) 150^0
2) 30^0
3) 60^0
4) 120^0

20. If $\vec{a}$ is vector perpendicular to both $\vec{b}$ and $\vec{c}$, then

1) $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = 0$
2) $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{0}$
3) $\vec{a} \times \left(\vec{b} + \vec{c}\right) = \vec{0}$
4) $\vec{a} + \left(\vec{b} + \vec{c}\right) = \vec{0}$

(Space for Rough Work)

21. If the area of the parallelogram with $\vec{a}$ and $\vec{b}$ as two adjacent sides is 15 sq. units, then the area of the parallelogram having $3\vec{a}+2\vec{b}$ and $\vec{a}+3\vec{b}$ as two adjacent sides in sq. units is

1) 45
2) 75
3) 105
4) 120

22. The locus of the point which moves such that the ratio of its distances from two fixed points in the plane is always a constant $K\ (<\ 1)$ is

1) circle
2) straight line
3) ellipse
4) hyperbola

23. If the lines $x+3y-9=0$, $4x+by-2=0$ and $2x-y-4=0$ are concurrent, then $b=$

1) 0
2) 1
3) 5
4) -5

24. The lines represented by $ax^2+2hxy+by^2=0$ are perpendicular to each other if

1) $h=0$
2) $h^2=ab$
3) $a+b=0$
4) $h^2=a+b$

25. The equation of the circle having $x-y-2=0$ and $x-y+2=0$ as two tangents and $x+y=0$ as a diameter is

1) $x^2+y^2=1$
2) $x^2+y^2=2$
3) $x^2+y^2-2x+2y-1=0$
4) $x^2+y^2+2x-2y+1=0$

(Space for Rough Work)

26. If the length of the tangent from any point on the circle $(x-3)^2+(y+2)^2=5r^2$ to the circle $(x-3)^2+(y+2)^2=r^2$ is 16 units, then the area between the two circles in sq. units is

1) 16π
2) 8π
3) 4π
4) 32π

27. The circles $ax^2+ay^2+2g_1x+2f_1y+c_1=0$ and $bx^2+by^2+2g_2x+2f_2y+c_2=0$ $(a\neq 0$ and $b\neq 0)$ cut orthogonally if

1) $g_1g_2+f_1f_2=c_1+c_2$
2) $bg_1g_2+af_1f_2=bc_1+ac_2$
3) $g_1g_2+f_1f_2=bc_1+ac_2$
4) $g_1g_2+f_1f_2=ac_1+bc_2$

28. The equation of the common tangent of the two touching circles, $y^2+x^2-6x-12y+37=0$ and $x^2+y^2-6y+7=0$ is

1) $x+y+5=0$
2) $x+y-5=0$
3) $x-y+5=0$
4) $x-y-5=0$

29. The equation of the parabola with vertex at $(-1,1)$ and focus $(2,1)$ is

1) $y^2-2y-12x+13=0$
2) $y^2-2y+12x+11=0$
3) $x^2+2x-12y+13=0$
4) $y^2-2y-12x-11=0$

30. The equation of the line which is tangent to both the circle $x^2+y^2=5$ and the parabola $y^2=40x$ is

1) $2x+y+5=0$
2) $2x-y-5=0$
3) $2x-y+5=0$
4) $2x-y\pm 5=0$

(Space for Rough Work)

31. $x = 4(1 + Cos\theta)$ and $y = 3(1 + Sin\theta)$ are the parametric equations of

1) $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$
2) $\frac{(x-4)^2}{16} - \frac{(y-3)^2}{9} = 1$
3) $\frac{(x+4)^2}{16} + \frac{(y+3)^2}{9} = 1$
4) $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 1$

32. If the distance between the foci and the distance between the directrices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3 : 2 , then $a : b$ is =

1) 2 : 1
2) 1 : 2
3) $\sqrt{3} : \sqrt{2}$
4) $\sqrt{2} : 1$

33. The ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ have in common

1) centre and vertices only
2) centre, foci and vertices
3) centre, foci and directrices
4) centre only

34. If $Sec\theta = m$ and $Tan\theta = n$, then $\frac{1}{m}\left[(m+n) + \frac{1}{(m+n)}\right] =$

1) mn
2) $2n$
3) $2m$
4) 2

35. The value of $\frac{Sin\,85^0 - Sin\,15^0}{Cos\,65^0} =$

1) 0
2) 1
3) -1
4) 2

(Space for Rough Work)

36. From an aeroplane flying, vertically above a horizontal road, the angles of depression of two consecutive stones on the same side of the aeroplane are observed to be 30^0 and 60^0 respectively. The height at which the aeroplane is flying in km is

1) 2
2) $\frac{2}{\sqrt{3}}$
3) $\frac{\sqrt{3}}{2}$
4) $\frac{4}{\sqrt{3}}$

37. If the angles of a triangle are in the ratio 3 : 4 : 5, then the sides are in the ratio

1) $3:4:5$
2) $2:\sqrt{3}:\sqrt{3}+1$
3) $\sqrt{2}:\sqrt{6}:\sqrt{3}+1$
4) $2:\sqrt{6}:\sqrt{3}+1$

38. If $Cos^{-1}x=\alpha$, $(0<x<1)$ and $Sin^{-1}\left(2x\sqrt{1-x^2}\right)+Sec^{-1}\left(\frac{1}{2x^2-1}\right)=\frac{2\pi}{3}$,

then $Tan^{-1}(2x)=$

1) $\frac{\pi}{2}$
2) $\frac{\pi}{3}$
3) $\frac{\pi}{4}$
4) $\frac{\pi}{6}$

39. If $a>b>0$, then the value of $Tan^{-1}\left(\frac{a}{b}\right)+Tan^{-1}\left(\frac{a+b}{a-b}\right)$ depends on

1) neither a nor b
2) a and not b
3) b and not a
4) both a and b

40. Which one of the following equations has no solution ?

1) $\sqrt{3}\,Sin\,\theta-Cos\,\theta=2$
2) $Cos\,\theta+Sin\,\theta=\sqrt{2}$
3) $Cosec\,\theta\cdot Sec\,\theta=1$
4) $Cosec\,\theta-Sec\,\theta=Cosec\,\theta\cdot Sec\,\theta$

(Space for Rough Work)

41. The complex number $\dfrac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)}$ when represented in the Argand diagram lies

1) on the X-axis (Real axis)
2) on the Y-axis (Imaginary axis)
3) in the first quadrant
4) in the second quadrant

42. If $2x=-1+\sqrt{3}i$, then the value of $(1-x^2+x)^6-(1-x+x^2)^6=$

1) 0
2) 64
3) -64
4) 32

43. The modulus and amplitude of $(1+i\sqrt{3})^8$ are respectively

1) 256 and $\dfrac{8\pi}{3}$
2) 2 and $\dfrac{2\pi}{3}$
3) 256 and $\dfrac{2\pi}{3}$
4) 256 and $\dfrac{\pi}{3}$

44. The value of $\underset{x\to 0}{Limit}\ \dfrac{5^x-5^{-x}}{2x}=$

1) $2\,Log\,5$
2) 1
3) 0
4) $Log\,5$

45. Which one of the following is not true always ?

1) If a function $f(x)$ is continuous at $x=a$, then $\underset{x\to a}{Limit}\ f(x)$ exists.
2) If $f(x)$ and $g(x)$ are differentiable at $x=a$, then $f(x)+g(x)$ is also differentiable at $x=a$
3) If $f(x)$ is continuous at $x=a$, then it is differentiable at $x=a$
4) If $f(x)$ is not continuous at $x=a$, then it is not differentiable at $x=a$.

(Space for Rough Work)

46. If $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots\ldots$ to ∞ with $|x| > 1$ then $\frac{dy}{dx} =$

1) $\frac{-y^2}{x^2}$
2) $\frac{y^2}{x^2}$
3) $x^2 y^2$
4) $\frac{x^2}{y^2}$

47. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then $f'(x) =$

1) $3x^2 + \frac{3}{x^4}$
2) $1 + \frac{1}{x^2}$
3) $x^2 - \frac{1}{x^2}$
4) $3x^2 - 3$

48. The derivative of $a^{Sec\,x}$ w.r.t. $a^{Tan\,x}$ $(a > 0)$ is

1) $a^{Sec\,x - Tan\,x}$
2) $Sin\,x\ a^{Sec\,x - Tan\,x}$
3) $Sin\,x\ a^{Tan\,x - Sec\,x}$
4) $Sec\,x\ a^{Sec\,x - Tan\,x}$

49. If $Sin(x+y) + Cos(x+y) = Log(x+y)$, then $\frac{d^2y}{dx^2} =$

1) 1
2) -1
3) 0
4) $\frac{-y}{x}$

50. If $f(x)$ is a function such that $f''(x) + f(x) = 0$ and $g(x) = [f(x)]^2 + [f'(x)]^2$ and $g(3) = 8$, then $g(8) =$

1) 8
2) 3
3) 0
4) 5

(Space for Rough Work)

51. If the curve $y = 2x^3 + ax^2 + bx + c$ passes through the origin and the tangents drawn to it at $x = -1$ and $x = 2$ are parallel to the X-axis, then the values of a, b and c are respectively.

1) 3, – 12 and 0
2) – 3, 12 and 0
3) – 3, – 12 and 0
4) 12, – 3 and 0

52. A circular sector of perimeter 60 metre with maximum area is to be constructed. The radius of the circular arc in metre must be

1) 10
2) 15
3) 5
4) 20

53. The tangent and the normal drawn to the curve $y = x^2 - x + 4$ at $P(1, 4)$ cut the X-axis at A and B respectively. If the length of the subtangent drawn to the curve at P is equal to the length of the subnormal, then the area of the triangle PAB in sq. units is

1) 16
2) 8
3) 32
4) 4

54. $\int \frac{(x^3 + 3x^2 + 3x + 1)}{(x+1)^5} dx =$

1) $Tan^{-1}x + c$
2) $Log(x+1) + c$
3) $\frac{1}{5} Log(x+1) + c$
4) $-\frac{1}{(x+1)} + c$

55. $\int \frac{Cosec x}{Cos^2\left(1 + Log\, Tan\frac{x}{2}\right)} dx =$

1) $-Tan\left[1 + Log\, Tan\, x/2\right] + c$
2) $Sec^2\left[1 + Log\, Tan\, x/2\right] + c$
3) $Tan\left[1 + Log\, Tan\, x/2\right] + c$
4) $Sin^2\left[1 + Log\, Tan\, x/2\right] + c$

(Space for Rough Work)

56. $\int \frac{dx}{x\sqrt{x^6 - 16}} =$

1) $Sec^{-1}\left(\frac{x^3}{4}\right) + c$
2) $\frac{1}{12} Sec^{-1}\left(\frac{x^3}{4}\right) + c$
3) $Cosh^{-1}\left(\frac{x^3}{4}\right) + c$
4) $\frac{1}{3} Sec^{-1}\left(\frac{x^3}{4}\right) + c$

57. If $I_1 = \int_0^{\pi/2} x \, Sin \, x \, dx$ and $I_2 = \int_0^{\pi/2} x \, Cos \, x \, dx$, then which one of the following is true ?

1) $I_1 = I_2$
2) $I_1 + I_2 = 0$
3) $I_1 = \frac{\pi}{2} I_2$
4) $I_1 + I_2 = \frac{\pi}{2}$

58. If $f(x)$ is defined in $[-2, 2]$ by $f(x) = 4x^2 - 3x + 1$ and $g(x) = \frac{f(-x) - f(x)}{(x^2 + 3)}$, then

$\int_{-2}^{2} g(x) dx =$

1) 24
2) 0
3) -48
4) 64

59. The area enclosed between the parabola $y = x^2 - x + 2$ and the line $y = x + 2$ in sq. units =

1) $\frac{4}{3}$
2) $\frac{2}{3}$
3) $\frac{1}{3}$
4) $\frac{8}{3}$

60. The solution of the differential equation $e^{-x}(y+1)\,dy + \left(Cos^2 x - Sin\,2x\right)y\,(dx) = 0$ subjected to the condition that $y = 1$ when $x = 0$ is

1) $(y+1) + e^x Cos^2 x = 2$
2) $y + Log\, y = e^x Cos^2 x$
3) $Log\,(y+1) + e^x Cos^2 x = 1$
4) $y + Log\, y + e^x Cos^2 x = 2$

(Space for Rough Work)