MATHEMATICS

Test Admission Ticket No.

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1. If $\vec{u}=\vec{a}-\vec{b}$, $\vec{v}=\vec{a}+\vec{b}$, and $|\vec{a}|=|\vec{b}|=2$, then $|\vec{u}\times\vec{v}|$ is

a) $2\sqrt{16-(\vec{a}\cdot\vec{b})^2}$

b) $2\sqrt{4-(\vec{a}\cdot\vec{b})^2}$

c) $\sqrt{16-(\vec{a}\cdot\vec{b})^2}$

d) $\sqrt{4-(\vec{a}\cdot\vec{b})^2}$

$|\vec{u}\times\vec{v}| = |(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})| = 2\vec{a}\times\vec{b} = 2|\vec{a}||\vec{b}|\sin\theta$

$= 2|\vec{a}||\vec{b}|\sqrt{1-\cos^2\theta}$

$= 2\sqrt{|\vec{a}|^2|\vec{b}|^2-|\vec{a}||\vec{b}|\cos^2\theta}$

$= 2\sqrt{4\times 4-(\vec{a}\cdot\vec{b})^2}$

$= 2\sqrt{16-(\vec{a}\cdot\vec{b})^2}$ a

2. The volume of the tetrahedron formed by the points (1, 1, 1), (2, 1, 3) (3, 2, 2) and (3, 3, 4) in cubic units is

a) $\frac{5}{6}$

b) $\frac{6}{5}$

c) 5

d) $\frac{2}{3}$

A B C D

$\vec{AB} = \hat{i}+2\hat{k}$, $\vec{AC} = 2\vec{i}+\vec{j}+\vec{k}$, $\vec{AD} = 2\vec{i}+2\vec{j}+3\vec{k}$

Volume of the tetrahedron

$= \frac{1}{6}[\vec{AB}\times\vec{AC}\cdot\vec{AD}] = 5/6$ a

3. Unit vector perpendicular to $\hat{i}-2\hat{j}+2\hat{k}$ and lying in the plane containing $\hat{i}+\hat{j}-2\hat{k}$ and $-\hat{i}+2\hat{j}+\hat{k}$ is

a) $8\hat{i}-7\hat{j}+11\hat{k}$

b) $8\hat{i}+7\hat{j}-11\hat{k}$

c) $8\hat{i}-7\hat{j}-11\hat{k}$

d) $\frac{1}{\sqrt{234}}(8\hat{i}-7\hat{j}-11\hat{k})$

only $\frac{1}{\sqrt{234}}(8\hat{i}-7\hat{j}-11\hat{k})$ is an unit vector and $\perp^{ar}$ to $\hat{i}-2\hat{j}+2\hat{k}$

d

(Space for calculation / rough work)

Sway.

$\hat{i}-2\hat{j}+2\hat{k}$

4. In the group $Q - \{-1\}$ under the binary operation $*$ defined by $a * b = a + b + ab$ the inverse of 10 is

a) $\frac{1}{10}$

b) $\frac{11}{10}$

d

c) $\frac{-11}{10}$

d) $\frac{-10}{11}$

5. In the group $\{1, 2, 3, 4, 5, 6\}$ under multiplication mod 7, $2^{-1} \times 4 =$

a) 1

c

b) 4

c) 2

d) 3

6. The group $(Z, +)$ has

a) exactly one subgroup

b) only two subgroups

d infinitely many subgroup.

c) no subgroups

d) infinitely many subgroups

Space for calculation / rough work

7. If 3x

a)

b)

c)

d)

8. The

a)

b)

c)

d)

9. The

a)

b)

c)

d)

7. If $3x \equiv 5 \pmod 7$, then

$3x \equiv 5 \pmod 7$, then $x = 4$

a) $x \equiv 2 \pmod 7$

Ans :-

$x \equiv 4 \pmod 7$

b) $x \equiv 3 \pmod 7$

c

c) $x \equiv 4 \pmod 7$

d) none of these

8. The argument of the complex number $\sin\left(\frac{6\pi}{5}\right) + i\left(1 + \cos\frac{6\pi}{5}\right)$ is

c

a) $\frac{\pi}{10}$

b) $\frac{5\pi}{6}$

c) $\frac{-\pi}{10}$

d) $\frac{2\pi}{5}$

9. The maximum value of $n < 101$ such that $1 + \sum_{k=1}^{n} i^k = 0$ is

c

a) 96

b) 97

c) 99

d) 100

Space for calculation / rough work

10. The value of $\left(-1+\sqrt{-3}\right)^{62}+\left(-1-\sqrt{-3}\right)^{62}$ is

a) 2^{62}

b) 2^{63}

c) -2^{62}

d) 0

$$2^{62}\left[\left(\frac{-1+\sqrt{-3}}{2}\right)^{62}+\left(\frac{-1-\sqrt{-3}}{2}\right)^{62}\right]$$

$$2^{62}\{\omega^{62}+\omega^{124}\} = 2^{62}\{\omega^2+\omega\} = 2^{62}(-1)$$

$$= \underline{\underline{-2^{62}}}$$

11. All complex numbers z which satisfy the equation $\left|\frac{z-6i}{z+6i}\right|=1$ lie on the

a) imaginary axis

b) real axis

c) neither of the axes

d) none of these

$$\left|\frac{(x+iy)-6i}{x+iy+6i}\right|=1 \text{ or, } \frac{(x+iy)-6i}{(x+iy+6i)} \times \frac{x-iy-6i}{x-i(y+6)}=1$$

$$= \frac{(y^2+x^2+12y-36)+i12x}{x^2+(y+6)^2}$$

Solve $\underline{\text{it}}$

12. The value of $\sin\left[\cot^{-1}\left\{\cos\left(\tan^{-1}x\right)\right\}\right]$ is

a) $\left(\sqrt{\frac{1+x^2}{2+x^2}}\right)$

b) $\left(\sqrt{\frac{2+x^2}{1+x^2}}\right)$

c) $\left(\sqrt{\frac{x^2-2}{x^2-1}}\right)$

d) $\left(\sqrt{\frac{x^2-1}{x^2-2}}\right)$

$$= \sin\left[\cot^{-1}\left\{\cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}\right]$$

$$= \sin\left[\cot^{-1}\left\{1/\sqrt{1+x^2}\right)\right]$$

$$= \underline{\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}}$$

13. The value of $\alpha(\neq 0)$ for which the function $f(x)=1+\alpha x$ is the inverse of itself is

Let $y=f(x)$, $y=1+\alpha x \Rightarrow x=\frac{y-1}{\alpha}$

$f(x)$ is the inverse of it self

$\frac{x-1}{\alpha}=(1+\alpha x)$

or, $(\alpha^2-1)x+(\alpha+1)=0$

$(\alpha+1)\{\alpha x-x+1\}=0$

$\alpha=-1$

a) -2

b) 2

c) -1

d) 1

C

14. If x^r occurs is the expansion of $\left(x+\frac{1}{x}\right)^n$, then its coefficient is

K_{th} term $= {}^nC_K\, x^K\left(\frac{1}{x}\right)^{n-K}$; coefficient of x^{2K-n}

Power of x; x^{2K-n}. ${}^nC_K$

Let $x^{2K-n}\equiv x^r$;

$r=2K-n \Rightarrow K=\frac{n+r}{2}$

Now ${}^nC_K={}^nC_{\frac{n+r}{2}}=\frac{n!}{\frac{n+r}{2}!\ \frac{n-r}{2}!}$

a) $\frac{n!}{(r!)^2}$

b) $\frac{n!}{(r+1)!\,(r-1)!}$

c) $\frac{n!}{\left(\frac{n+r}{2}\right)!\left(\frac{n-r}{2}\right)!}$

d) $\frac{n!}{\left[\left(\frac{r}{2}\right)!\right]^2}$

C

15. If $\tan A-\tan B=x$ and $\cot B-\cot A=y$ then $\cot(A-B)=$

$\cot(A-B)=\frac{1+\tan A\cdot\tan B}{\tan A-\tan B}$ — ①

$\frac{1}{\tan B}-\frac{1}{\tan A}=\frac{\tan A-\tan B}{\tan A\cdot\tan B}=y$ — ②

Given. $\tan-\tan B=x$ — ③

Eq ② + 3 ÷ $\tan A\cdot\tan B=x/y$, put this value in eqn ①

$\cot(A-B)=\frac{1+x/y}{x}=\frac{1}{x}+\frac{1}{y}$

a) $\frac{1}{y}-\frac{1}{x}$

b) $\frac{1}{x}-\frac{1}{y}$

c) $\frac{1}{x}+\frac{1}{y}$

d) none of these

C

Space for calculation / rough work

16. $\cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \cos^2\frac{5\pi}{12} =$

$\cos^2 15^\circ + \frac{1}{2} + \cos^2 75^\circ$

$= \cos^2 15^\circ + 1 - \sin^2 75^\circ + 1/2$

$= \cos^2 15^\circ - \sin^2 75^\circ + 3/2$

$= \cos^2 15^\circ - \cos^2(90^\circ - 15^\circ) + 3/2$

$= \frac{3}{2} + \cos^2 15^\circ - \cos^2 15^\circ$

$= 3/2$

a) $\frac{3}{2}$ ✓

b) $\frac{3-\sqrt{3}}{2}$

c) $\frac{2}{3}$

d) $\frac{2}{3+\sqrt{3}}$

(a)

17. If $\sin\theta$, $\cos\theta$ and $\tan\theta$ are in G.P. then $\cot^6\theta - \cot^2\theta$ is

a) 1 ✓

b) $\frac{1}{2}$

c) 2

d) 3

(a)

$\cos^2\theta = \sin\theta \cdot \tan\theta \Rightarrow \cos^3\theta = \sin^2\theta$

or, $\cos^3\theta = 1 - \cos^2\theta$ or, $\cos^3\theta + \cos^2\theta - 1 = 0$

solve it for θ and replace in $\cot^6\theta - \cot^2\theta = 1$

18. If $\frac{3x^2 - 2x + 4}{(x-1)^6} = \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2} + \frac{A_3}{(x+1)^3} + \frac{A_4}{(x+1)^4} + \frac{A_5}{(x+1)^5} + \frac{A_6}{(x+1)^6}$, then

$(A_1 + A_3 + A_5,\ A_2 - A_4 - A_6) =$

a) (0,0)

b) (−8, −12)

c) (8, −12)

d) (−8, 12) ✓

(d)

Put $x = 0$,

then $A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 4$

only "d" (−8, 12) satisfies

the solution

Space for calculation / rough work

19. If $\log_2\left(2^{x-1}+6\right)+\log_2\left(4^{x-1}\right)=5$, then $x=$; $\log_2\left(2^{x-1}+6\right)\left(2^{2x-2}\right)=5$

or, $\left(2^{x-1}+6\right)\left(2^{2x-2}\right)=2^5$; Let $y=2^{x-1}$

$(y+6)y^2=32$ or $(y-2)(y^2+8y+16)=0$

Possible soln: $y=2^{x-1}=2^1 \therefore x-1=1$ or, $x=2$

a) 4
b) 1
c) 3
d) 2

d

20. If a,b,c,d are the roots of the equation $x^4+2x^3+3x^2+4x+5=0$, then $1+a^2+b^2+c^2+d^2$ is equal to

a) -2
b) -1
c) 2
d) 1

b

$1+(a^2+b^2+c^2+d^2)=1+(a+b+c+d)^2-2(ab+ac+ad+bc+bd+cd)$

$=1+(\text{sum of roots})^2-2(\text{sum of multiplication of roots})$

$=1+2^2-2\times3=5-6=-1$ Ans

21. If $C_0, C_1, C_2, \cdots, C_n$ are binomial coefficients of order n, then the value of $\frac{C_1}{2}+\frac{C_3}{4}+\frac{C_5}{6}+\cdots=$

a) $\frac{2^n+1}{n+1}$

b) $\frac{2^n-1}{n+1}$

c) $\frac{2^n+1}{n-1}$

d) $\frac{2^n}{n+1}$

b

$(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n$

Integrate both side from 0 to 1:

$\frac{2^n-1}{n+1}=C_0+C_1/2+C_2/3+\ldots+C_n/n+1$ — (1)

again $(1-x)^n=C_0-C_1x+C_2x^2+\ldots+(-1)^nC_nx^n$

Integrate both side 0 to 1 (:

$\frac{1}{n+1}=C_0-C_1/2+C_2/3-\ldots$ — (2) Perform (1) − (2)

result $\Rightarrow \frac{2^n-1}{n+1}=C_1/2+\frac{C_3}{4}+\frac{C_5}{6}+\ldots$

22. The value of $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\text{ to }\infty\right)}$ is

a) 4
b) $\frac{1}{4}$
c) 2
d) $\frac{1}{2}$

a

$(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\left(\frac{1}{2}\right)^2+\ldots\infty\right)}$

$=(0.2)^{\log_{\sqrt{5}}(1/4)/1-1/2}=(0.2)^{\log_{\sqrt{5}}(1/2)}$

$=(0.2)^{-1\log_{\sqrt{5}}2}$

$=\left(\frac{1}{5}\right)^{-\log_{\sqrt{5}}2}=$ ~~$\log_{\sqrt{5}}$~~ $5^{2\log_5 2}$

$=5^{\log_5 4}=4$

27. If $n(A) = n(B) = m$, then the number of possible bijections from A to B is

a) m

b) m^2

c) $m!$

d) $2m$

(c) $m!$

28. $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right] =$

a) $\sin^{-1}x - \sin^{-1}\sqrt{1-x}$

b) $\sin^{-1}x + \sin^{-1}\sqrt{1-x}$

c) $\sin^{-1}x - \sin^{-1}\sqrt{x}$

d) $\sin^{-1}x + \sin^{-1}\sqrt{x}$

(c)

$\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

$= \sin^{-1}\left[x\sqrt{1-(\sqrt{x})^2} - \sqrt{x}\sqrt{1-x^2}\right]$ — (1)

W.K.T. $\sin^{-1}(x+y) = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$ — (2)

Compare (1) & (2) :-

$\boxed{\sin^{-1}x - \sin^{-1}\sqrt{x}}$

29. If $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \tan 4\theta \tan 7\theta$, then the general solution is

a) $\theta = \frac{n\pi}{4}$

b) $\theta = \frac{n\pi}{12}$

c) $\theta = \frac{n\pi}{6}$

d) none of these

(b)

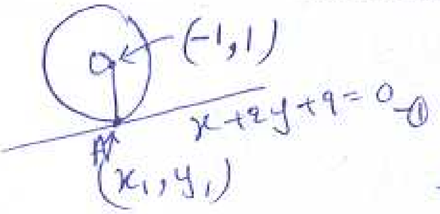
30. If a circle with the point $(-1, 1)$ as its center touches the straight line $x + 2y + 9 = 0$ then the coordinates of the points of contact is

a) $(-3, 3)$

b) $(-3, -3)$

c) $(0, 0)$

d) $\left(\frac{7}{3}, -\frac{17}{3}\right)$



$\perp^{ar}$ of equ^n of $x+2y+9=0$ is $y = 2x + c$ — (2)

Equ^n of OA $\Rightarrow$ $1 = -2 + c$;

$c = -3$

$y = 2x - 3$ — (3)

Solve equ^n (1) & (3) :-

Space for rough work

$x = -3$

$y = -3$

Answer — (b)

27. If the circles $x^2+y^2+2gx+2fy=0$, and $x^2+y^2+2g^1x+2f^1y=0$ touch each other, then

For given condition:- $\frac{2g}{2g'} = \frac{2f}{2f'} \Rightarrow \boxed{f'g = g'f}$

a) $fg = f^1g^1$
b) $f^1g = fg^1$
c) $ff^1 = gg^1$
d) none of these

(b)

28. The number of common tangents to the circles $x^2+y^2=4$ and $x^2+y^2-4x+2y-4=0$ is

a) 1
b) 2
c) 3
d) 4

(b)

Eqn are:- $x^2+y^2=(2)^2$; — ①
$(x-2)^2+(y+1)^2 = 3^2$ — ②
Cut each other at two places:- so, no. common tangents $=2$

29. The length of the tangent drawn from any point on the circle $x^2+y^2-4x+6y-4=0$ to the circle $x^2+y^2-4x+6y=0$ is

a) 8
b) 4
c) 2
d) none of these

(c)

Length of tangent from $x^2+y^2+2gx+2fy+c_1$ to circle $x^2+y^2+2gx+2fy+c_2=0$ is $\sqrt{c_2-c_1}$
here $c_2=4$, $c_1=0$
so, length $= \sqrt{4-0} = 2$

30. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then the value of b^2 is

a) 25
b) 9
c) 16
d) 4

(c)

For hyperbola:- $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$; eccentricity $e=\frac{225}{144}=15/12$
foci $\left(\pm\frac{12}{5}\times\frac{15}{12}, 0\right) = (\pm 3, 0)$
Since foci of eclipse coincide:-
$\Rightarrow 5e=3$ or, $e=3/5$
Since $b^2=a^2(1-e^2)$ or $b^2=25(1-9/25)=\boxed{16}$ Ans

31. The latus rectum of the ellipse is half the minor axis. Then its eccentricity is

a) $\frac{1}{\sqrt{2}}$
b) $\frac{1}{\sqrt{3}}$
c) $\frac{\sqrt{3}}{2}$
d) none of these

(c)

latus Rectum $= 2b^2/a$, $L_{\text{minor axis}} = 2b$
Given $\frac{2b}{2} = 2b^2/a \Rightarrow a=2b$
W.K.T. $b^2=a^2(1-e^2)$
$1-e^2=1/4$ or, $e^2=3/4$
$\boxed{e=\sqrt{3}/2}$

Space for calculation/rough work

32. The ends of the latus rectum of the parabola $x^2+10x-16y+25=0$ are

a) $(3,4),(-13,4)$

b) $(5,-8),(-5,8)$

c) $(3,-4),(13,4)$

d) $(-3,-4),(13,-4)$

(a)

$(x+5)^2=4(4)y$

$X^2=4aY$

vertex $\equiv(-5,0)$, focus $\equiv(-5,4)$

equ^n of axis $\Rightarrow x=4$

~~only (3,4~~ equ^n of a line $\perp^{ar}$ to axis and passing through focus is $y=4$; $(3,4),(-13,4)$

satisfies the parabola and their y-coordinate is 4

33. Which of the following functions is differentiable at $x=0$?

a) $\cos(|x|)+|x|$

b) $\cos(|x|)-|x|$

c) $\sin(|x|)+|x|$

d) $\sin(|x|)-|x|$

(d)

34. If $x=a\left(\cos t+\log\tan\frac{t}{2}\right), y=a\sin t$, then $\frac{dy}{dx}=$

a) $\tan t$

b) $\cot t$

c) $-\cot t$

d) $-\tan t$

(a)

Find $\frac{dy}{dt}$ and $\frac{dx}{dt}$

$$\frac{(dy/dt)}{(dx/dt)}=\frac{\cos t\sin t}{\cos^2 t}=\tan t$$

35. If $\begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}1 & \tan\theta\\ -\tan\theta & 1\end{bmatrix}^{-1}=\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$, then

a) $a=1=b$

b) $a=\cos 2\theta, b=\sin 2\theta$

c) $a=\sin 2\theta, b=\cos 2\theta$

d) $a=\cos\theta, b=\sin\theta$

(b)

$$\begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}\frac{1}{\sec^2\theta}$$

$$=\begin{bmatrix}1-\tan^2\theta & -2\tan\theta\\ 2\tan\theta & 1-\tan^2\theta\end{bmatrix}\frac{1}{\sec^2\theta}$$

$$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & -2\sin\theta\cos\theta\\ +2\sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$

So, $\left.\begin{matrix}a=\cos 2\theta\\ b=\sin 2\theta\end{matrix}\right\}$

Space for calculation / rough work

36. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n is

a) $\begin{bmatrix} 1 & 2^n - 2 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & n^2 \\ 1 & 1 \end{bmatrix}$

c $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

37. If α, β, ν are the roots of the equation $x^3 + px + q = 0$ then the value of the determinant $\begin{vmatrix} \alpha & \beta & \nu \\ \beta & \nu & \alpha \\ \nu & \alpha & \beta \end{vmatrix}$ is

a) q

b) 0

c) p

d) $p^2 - 2q$

b

38. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$ in the interval $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ is

a) 0

b) 1

c) 2

d) 3

b

Space for calculation / rough work

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Suraj

39. The sum of non-prime positive divisors of 450 is

a) 1209
b) 1299
c) 1199
d) 1099

c

40. The last digit of $\sum_{\substack{1<p<100 \\ p\,-\,\text{prime}}} p! - \sum_{n=1}^{50} (2n)!$ is

a) 2
b) 4
c) 6
d) 8

d

41. The interval I such that $\int_0^1 \frac{dx}{\sqrt{1+x^4}} \in$ I is given by

a) $\left(0, \frac{1}{\sqrt{2}}\right)$
b) $\left[\frac{1}{\sqrt{2}}, 1\right]$
c) $\left[\sqrt{2}, 2\right]$
d) $\left[\sqrt{2}, \frac{7}{4}\right]$

$(1+x^4) < (1+x^2)^2 \Rightarrow \sqrt{1+x^4} < 1+x^2$

or, $\frac{1}{\sqrt{1+x^4}} > \frac{1}{1+x^2}$ or, $\frac{1}{1+x^2} < \frac{1}{\sqrt{1+x^4}}$

$\frac{1}{\sqrt{1+x^4}} < 1$ always

So, $\int_0^1 \frac{1}{1+x^2}\,dx < \int_0^1 \frac{1}{\sqrt{1+x^4}}\,dx < \int_0^1 1\,dx$

b

42. $\int_0^{\frac{\pi}{2}} \log(\tan x)\ dx =$

a) $\frac{\pi}{2}$
b) 0
c) 1
d) $\frac{\pi}{4}$

b

$\int_0^{\pi/2} \log(\tan x) = \int_0^{\pi/2} \log \sin x - \int_0^{\pi/2} \log \cos x$

$= 0$

$\Rightarrow \int_0^{\pi/2} \log(\sin x) = \int_0^{\pi/2} \log(\cos x)$

Space for calculation / rough work

43. The value of $\int_{-2}^{2}(ax^3+bx+c)\,dx$ depends on the

Since ax^3+bx is an odd function $\int_{-2}^{2}(ax^3+bx)dx=0$

a) value of b

b) value of c

c) value of a

d) values of a and b

(b) then $\int_{-2}^{2}(ax^3+bx+c)dx = \int_{-2}^{2}c(dx)$; ∴ integral depends upon the value of 'c'

44. The area of the region bound by the curves $y = x^2$ and $y = 4x - x^2$ is

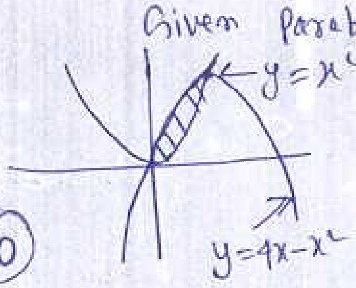
a) $\frac{16}{3}$ sq. units

b) $\frac{8}{3}$ sq. units

c) $\frac{4}{3}$ sq. units

d) $\frac{2}{3}$ sq. units

Given Parabolas are $y=x^2$, $(y-4)=-(x-2)^2$



x - coordinates on intersect Pt. = 0 or 2

(b)

area $=\int_0^2\{(4x-x^2)-x^2\}dx = \int_0^2(4x-2x^2)dx$

$\left[2x^2-\frac{2}{3}x^3\right]_0^2 = 8/3$

45. The particular solution of $\frac{y}{x}\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, when $x = 1$, $y = 2$ is

a) $5(1+y^2) = 2(1+x^2)$

b) $2(1+y^2) = 5(1+x^2)$

c) $5(1+y^2) = (1+x^2)$

d) $(1+y^2) = 2(1+x^2)$

(b)

$\frac{1}{2}\int\frac{2y\,dy}{1+y^2} = \frac{1}{2}\int\frac{2x\,dx}{1+x^2}$; or, $\frac{1}{2}\log(1+x^2) = \frac{1}{2}\log(1+y^2)+C$ —①

or, $\log\frac{(1+x^2)}{(1+y^2)} = C$

Put $x=1, y=2$

$C = \log 2/5$

Put value of C in equ ① : $2(1+y^2)=5(1+x^2)$

46. The solution of the differential equation $\frac{dy}{dx} = (x+y)^2$ is

a) $\frac{1}{x+y} = c$

b) $\sin^{-1}(x+y) = x + c$

c) $\tan^{-1}(x+y) = c$

d) $\tan^{-1}(x+y) = x + c$

(d)

Put $x+y=z \Rightarrow \frac{dy}{dx}+1=\frac{dz}{dx}$

Now given equ

$\frac{dz}{dx}-1 = z^2$ or, $\frac{dz}{dx}=1+z^2$

$\int dx = \int\frac{dz}{1+z^2}$

Space for calculation / rough work

$\boxed{c+x = \tan^{-1}(x+y)}$

47. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

a) $e^{1/2}$ (b)

b) $\sqrt{e}$

c) 1

d) e^2

48. Let x be a number which exceeds its square by the greatest possible quantity, then $x =$

a) 1/2 (a)

b) 1/4

c) 3/4

d) 1/3

Go by option. For $x=\frac{1}{2}$: $\frac{1}{2}-\left(\frac{1}{2}\right)^2=\frac{1}{4}$

49. The subtangent at $x = \pi/2$ on the curve $y = x \sin x$ is

a) 0 (c)

b) 1

c) $\pi/2$

d) none of these

slope $\left.\frac{dy}{dx}\right|_{x=\pi/2} = 1$; $y = x + c$; $P(\pi/2, 0)$ lies on $y = x\sin x$; so $c = \pi/2$

equ^n of line $\Rightarrow y = x + \pi/2$ (distance b/w origin and axis intersect pt $= \pi/2$)

50. The value of $\int \frac{10^{x/2}}{\sqrt{10^{-x} - 10^x}}\,dx$ is

a) $\frac{1}{\log_e 10}\sin^{-1}(10^x) + c$ (a)

b) $2\sqrt{10^{-x} + 10^x} + c$

c) $\frac{1}{\log_e 10}\sinh^{-1}(10^x) + c$

d) $\frac{-1}{\log_e 10}\sinh^{-1}(10^x) + c$

$$\int \frac{10^{x/2}\,dx}{\sqrt{10^{-x} - 10^x}} = \int \frac{10^{x/2}\,10^{x/2}}{\sqrt{1-(10^x)^2}}\,dx$$

$$= \int \frac{10^x}{\sqrt{1-(10^x)^2}}\,dx \;;\; y = 10^x = e^{x\log_e 10}$$

$$\frac{dy}{dx} = (\log_e 10)\, e^{x\log_e 10}$$

$$= \int \frac{\log_e 10\,(e^{x\log 10})}{\log_e 10\sqrt{1-(10^x)^2}}\,dx$$

$$= \frac{1}{\log_e 10}\int \frac{dy}{\sqrt{1-y^2}} \;;\; y = 10^x = e^{x\log_e 10}$$

$$= \frac{1}{\log 10}\sin^{-1}(10^x) + c$$

Space for calculation / rough work

51. $\int e^x \left\{\frac{1+\sin x \cos x}{\cos^2 x}\right\} dx =$

$\int e^x \left\{ \frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2} \right\} dx$

a) $e^x \cos x + c$

b) $e^x \sec x \tan x + c$ (c)

$\int e^x (\sec^2 x + \tan x)\,dx \equiv \int e^x (f'(x) + f(x))\,dx$

c) $e^x \tan x + c$

$= e^x \tan x + c$

d) $e^x \cos^2 x - 1 + c$

52. $\int \frac{x^2+1}{x^4+1} dx$ (d)

a) $\frac{1}{\sqrt{2}} \log_e (x^2+1) + c$

b) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2+1}{x\sqrt{2}}\right) + c$

c) $\frac{1}{\sqrt{2}} \tan^{-1}(x^2-1) + c$

d) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right) + c$

53. The locus of the mid point of the intercept of the line $x \cos\alpha + y \sin\alpha = p$ between the coordinate axes is

a) $x^{-2} + y^{-2} = 4p^{-2}$ (a)

b) $x^{-2} + y^{-2} = p^{-2}$

c) $x^2 + y^2 = 4p^{-2}$

d) $x^2 + y^2 = p^2$

when $x = 0$, $y = p \operatorname{cosec}\alpha$

$y = 0$, $x = p \sec\alpha$

mid point $\equiv \left(\frac{p \sec\alpha}{2}, \frac{p \operatorname{cosec}\alpha}{2}\right)$

so, $x = \frac{p \sec\alpha}{2}$; $y = \frac{p \operatorname{cosec}\alpha}{2}$

so, $\cos\alpha = \frac{p}{2x}$; $\sin\alpha = \frac{p}{2y}$

W.K.T

$\cos^2\alpha + \sin^2\alpha = 1$

Space for calculation / rough work

$\frac{p^2}{4x^2} + \frac{p^2}{4y^2} = 1$

$\boxed{x^{-2} + y^{-2} = 4p^{-2}}$

54. If the line through $A=(4,-5)$ is inclined at an angle 45^0 with the positive direction of the x-axis, then the coordinates of the two points on opposite sides of A at a distance of $3\sqrt{2}$ units are

a) $(7,2),(1,8)$

b) $(7,2),(1,-8)$

c) $(7,-2),(1,-8)$ ✓

d) $(7,2),(-1,8)$

Slope $= \tan 45^\circ = 1$; eqn $y = x + c$

$P(4,5)$ lies on line so, $c = -9$

Now, eqn $\Rightarrow y = x - 9$

only $\underline{(7,-2)}$ and $\underline{(1,-8)}$ lies on above st. line eqn, so no need for further calculation

C

55. If the line $px+qy=0$ coincides with one of the lines given by $ax^2+2hxy+by^2=0$ then

a) $ap^2+2hpq+bq^2=0$

b) $aq^2+2hpq+bp^2=0$

c) $aq^2-2hpq+bp^2=0$ ✓

d) none of these

$y = \frac{-p}{q}x$; put in given pair of lines

or, $ax^2 + 2hx\left(\frac{-px}{q}\right) + \frac{bp^2}{q^2}x^2 = 0$

or, $(aq^2 - 2hpq + bp^2)x^2 = 0$

Soln:- either $x = 0$ or $\boxed{aq^2 - 2hpq + bp^2 = 0}$

C

56. The function $f(x)=\left(\frac{\log_e(1+ax)-\log_e(1-bx)}{x}\right)$ is undefined at $x=0$. The value which should be assigned to f at $x=0$ so that it is continuous at $x=0$ is

a) $\frac{a+b}{2}$

b) $a+b$ ✓

c) $\log_e(a\,b)$

d) $a-b$

b

Ver A Mathe

57. lim

a)

b)

c)

d)

58. The n

a) 3

b) 2

c) 1

d) 6

59. The a

a) 17

b) 14

c) 13

d) 12

Space for calculation / rough work

coordi-

57. $\lim_{n\to\infty}\frac{(1^2+2^2+\cdots+n^2)\sqrt{n}}{(n+1)(n+10)(n+100)}=$

a) 3

b) $\frac{1}{3}$

c) $\frac{2}{3}$

d) ∞

(b)

$$\left(\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6(n+1)(n+10)(n+100)}\right)\left(\lim_{n\to\infty}\cdot\sqrt{n}\right)$$

$$=\lim_{n\to\infty}\frac{2n^2+n}{6(n+10)(n+100)}(1)=\lim_{n\to\infty}\frac{2+1/n}{6(1+10/n)(1+100/n)}$$

$$=\frac{2+0}{6(1+0)(1+0)}=\frac{2}{6}=1/3$$

58. The number of triangles in a complete graph with 10 non-collinear vertices is

a) 360

b) 240

c) 120

d) 60

(c)

no. of triangle $= {}^{10}C_3 = \frac{10\times9\times8\times 7!}{7!\times 3!} = 120$

59. The angle between hands of a clock when the time is 4.25 AM is

igned to

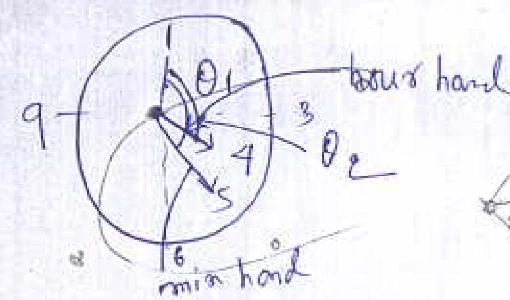
a) $17\frac{1}{2}^{0}$

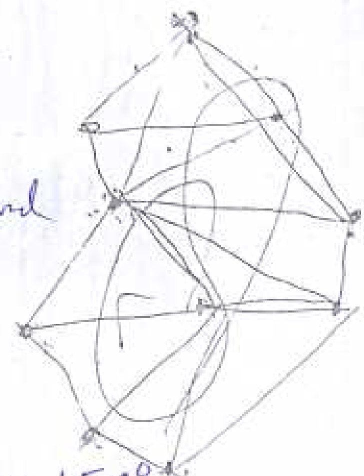
b) $14\frac{1}{2}^{0}$

c) $13\frac{1}{2}^{0}$

d) $12\frac{1}{2}^{0}$

(a)





θ_1 min hand $=\frac{360^\circ}{12}\times5=150^\circ$

θ_2 hour hand $=\frac{360^\circ}{12}\times4+\frac{30^\circ}{60\text{min}}\times25\text{ min}=132\cdot5^\circ$

$\theta_1-\theta_2=150-132.5^\circ=17\frac{1}{2}$

Space for calculation / rough work