WARNING:	Any malpractice or any attempt to commit any kind of malpractice in the Examination will DISQUALIFY THE CANDIDATE.

PAPER-II MATHEMATICS-2018

Version Code	B1	Question Booklet Serial Number :	4131000

Time: 150 Minutes	Number of Questions: 120	Maximum Marks: 480

Name of the Candidate	
Roll Number	
Signature of the Candidate	

INSTRUCTIONS TO CANDIDATES

1. **Please ensure that the version code shown at the top of this Question Booklet is same as that shown in the OMR Answer Sheet issued to you.** If you have received a Question Booklet with a different Version Code, please get it replaced with a Question Booklet with the same Version Code as that of OMR Answer Sheet from the Invigilator. **THIS IS VERY IMPORTANT.**
2. Please fill the items such as Name, Roll Number and Signature in the columns given above. Please also write Question Booklet Serial Number given at the top of this page against item 3 in the OMR Answer Sheet.
3. This Question Booklet contains 120 questions. For each question five answers are suggested and given against (A), (B), (C), (D) and (E) of which only one will be the **'Most Appropriate Answer.'** Mark the bubble containing the letter corresponding to the 'Most Appropriate Answer' in the OMR Answer Sheet, by using either **Blue or Black Ball Point Pen only.**
4. Negative Marking: In order to discourage wild guessing the score will be subjected to penalization formula based on the number of right answers actually marked and the number of wrong answer marked. Each correct answer will be awarded FOUR marks. ONE mark will be deducted for each incorrect answer. More than one answer marked against a question will be deemed as incorrect answer and will be negatively marked.
5. Please read the instructions in the OMR Answer Sheet for marking the answers. Candidates are advised to strictly follow the instruction contained in the OMR Answer Sheet.

IMMEDIATELY AFTER OPENING THE QUESTION BOOKLET, THE CANDIDATE SHOULD VERIFY WHETHER THE QUESTION BOOKLET CONTAINS ALL THE 120 QUESTIONS IN SERIAL ORDER. IF NOT, REQUEST FOR REPLACEMENT.

DO NOT OPEN THE SEAL UNTIL THE INVIGILATOR ASKS YOU TO DO SO.

SEAL

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PLEASE ENSURE THAT THIS QUESTION BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120.
PRINTED PAGES 32.

1. The value of $\frac{2(\cos 75^\circ + i \sin 75^\circ)}{0.2(\cos 30^\circ + i \sin 30^\circ)}$ is

(A) $\frac{5}{\sqrt{2}}(1+i)$ (B) $\frac{10}{\sqrt{2}}(1+i)$ (C) $\frac{10}{\sqrt{2}}(1-i)$ (D) $\frac{5}{\sqrt{2}}(1-i)$ (E) $\frac{1}{\sqrt{2}}(1+i)$

2. If the conjugate of a complex number z is $\frac{1}{i-1}$, then z is

(A) $\frac{1}{i-1}$ (B) $\frac{1}{i+1}$ (C) $\frac{-1}{i-1}$ (D) $\frac{-1}{i+1}$ (E) $\frac{1}{i}$

3. The value of $\left(i^{18} + \left(\frac{1}{i}\right)^{25}\right)^3$ is equal to

(A) $\frac{1+i}{2}$ (B) $2+2i$ (C) $\frac{1-i}{2}$ (D) $\sqrt{2} - \sqrt{2}i$ (E) $2-2i$

4. The modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ is

(A) 2 (B) $\sqrt{2}$ (C) 4 (D) 8 (E) 10

Space for rough work

5. If $z = e^{i4\pi/3}$, then $(z^{192} + z^{194})^3$ is equal to

(A) -2 (B) -1 (C) $-i$ (D) $-2i$ (E) 0

6. If a and b are real numbers and $(a+ib)^{11} = 1 + 3i$, then $(b+ia)^{11}$ is equal to

(A) $i + 3$ (B) $1 + 3i$ (C) $1 - 3i$ (D) 0 (E) $-i - 3$

7. If $\alpha \neq \beta$, $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $3x^2 - 19x - 3 = 0$ (B) $3x^2 + 19x - 3 = 0$

(C) $x^2 + 19x + 3 = 0$ (D) $3x^2 - 19x - 19 = 0$

(E) $3x^2 - 19x + 3 = 0$

8. The focus of the parabola $y^2 - 4y - x + 3 = 0$ is

(A) $\left(\frac{3}{4}, 2\right)$ (B) $\left(\frac{3}{4}, -2\right)$ (C) $\left(2, \frac{3}{4}\right)$ (D) $\left(\frac{-3}{4}, 2\right)$ (E) $\left(2, \frac{-3}{4}\right)$

Space for rough work

9. If $f: R \to (0, \infty)$ is an increasing function and if $\lim_{x \to 2018} \frac{f(3x)}{f(x)} = 1$, then $\lim_{x \to 2018} \frac{f(2x)}{f(x)}$ is equal to

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 2 (D) 3 (E) 1

10. If f is differentiable at $x = 1$, then $\lim_{x \to 1} \frac{x^2 f(1) - f(x)}{x-1}$ is

(A) $-f'(1)$ (B) $f(1) - f'(1)$ (C) $2f(1) - f'(1)$
(D) $2f(1) + f'(1)$ (E) $f(1) + f'(1)$

11. Eccentricity of the ellipse $4x^2 + y^2 - 8x + 4y - 8 = 0$ is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{4}$ (C) $\frac{\sqrt{3}}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{8}$ (E) $\frac{\sqrt{3}}{16}$

12. The focus of the parabola $(y + 1)^2 = -8(x + 2)$ is

(A) $(-4, -1)$ (B) $(-1, -4)$ (C) $(1,4)$ (D) $(4,1)$ (E) $(-1,4)$

Space for rough work

13. Which of the following is the equation of a hyperbola?

(A) $x^2 - 4x + 16y + 17 = 0$ (B) $4x^2 + 4y^2 - 16x + 4y - 60 = 0$

(C) $x^2 + 2y^2 + 4x + 2y - 27 = 0$ (D) $x^2 - y^2 + 3x - 2y - 43 = 0$

(E) $x^2 + 4x + 6y - 2 = 0$

14. Let $f(x) = px^2 + qx + r$, where p, q, r are constants and $p \neq 0$. If $f(5) = -3f(2)$ and $f(-4) = 0$, then the other root of f is

(A) 3 (B) −7 (C) −2 (D) 2 (E) 6

15. Let $f:\mathbb{R}\to\mathbb{R}$ satisfy $f(x)f(y) = f(xy)$ for all real numbers x and y. If $f(2) = 4$, then $f\left(\frac{1}{2}\right) =$

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 2

16. Sum of last 30 coefficients in the binomial expansion of $(1 + x)^{59}$ is

(A) 2^{29} (B) 2^{59} (C) 2^{58} (D) $2^{59} - 2^{29}$ (E) 2^{60}

Space for rough work

17. $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4 =$

(A) $20\sqrt{6}$ (B) $30\sqrt{6}$ (C) $5\sqrt{10}$ (D) $40\sqrt{6}$ (E) $10\sqrt{6}$

18. Three players A, B and C play a game.The probability that A, B and C will finish the game are respectively $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. The probability that the game is finished is

(A) $\frac{1}{8}$ (B) 1 (C) $\frac{1}{4}$ (D) $\frac{3}{4}$ (E) $\frac{1}{2}$

19. If $z_1 = 2 - i$ and $z_2 = 1 + i$, then $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$ is

(A) 2 (B) $\sqrt{2}$ (C) 3 (D) $\sqrt{3}$ (E) 1

20. If $f(x) = \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty} f(x)$ is equal to

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 0 (E) ∞

Space for rough work

21. The value of $\sin\frac{31}{3}\pi$ is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{-\sqrt{3}}{2}$ (D) $\frac{-1}{\sqrt{2}}$ (E) $\frac{1}{2}$

22. The sum of odd integers from 1 to 2001 is

(A) $(1121)^2$ (B) $(1101)^2$ (C) $(1001)^2$ (D) $(1021)^2$ (E) $(1011)^2$

23. If $y = \frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x}$, then $y'(x)$ is equal to

(A) $2\cos^2 x$ (B) $2\cos^3 x$ (C) $-\cos 2x$ (D) $\cos 2x$ (E) $3\cos x$

24. The foci of the hyperbola $16x^2 - 9y^2 - 64x + 18y - 90 = 0$ are

(A) $\left(\frac{24\pm5\sqrt{145}}{12}, 1\right)$ (B) $\left(\frac{21\pm5\sqrt{145}}{12}, 1\right)$ (C) $\left(1, \frac{24\pm5\sqrt{145}}{2}\right)$

(D) $\left(1, \frac{21\pm5\sqrt{145}}{2}\right)$ (E) $\left(\frac{21\pm5\sqrt{145}}{2}, -1\right)$

Space for rough work

25. If the sum of the coefficients in the expansion of $(a^2x^2 - 2ax + 1)^{51}$ is zero, then a is equal to

(A) 0 (B) 1 (C) −1 (D) −2 (E) 2

26. The mean deviation of the data 2,9,9,3,6,9,4 from the mean is

(A) 2.23 (B) 3.23 (C) 2.57 (D) 3.57 (E) 1.03

27. The mean and variance of a binomial distribution are 8 and 4 respectively. What is $(X = 1)$?

(A) $\frac{1}{2^8}$ (B) $\frac{1}{2^{12}}$ (C) $\frac{1}{2^6}$ (D) $\frac{1}{2^4}$ (E) $\frac{1}{2^5}$

28. The number of diagonals of a polygon with 15 sides is

(A) 90 (B) 45 (C) 60 (D) 70 (E) 10

Space for rough work

29. In a class, 40% of students study maths and science and 60% of students study maths. What is the probability of a student studying science given the student is already studying maths?

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{2}{3}$ (D) $\frac{1}{5}$ (E) $\frac{1}{4}$

30. The eccentricity of the conic $x^2 + 2y^2 - 2x + 3y + 2 = 0$ is

(A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\sqrt{2}$ (E) 1

31. If the mean of a set of observations $x_1, x_2, \ldots, x_{10}$ is 20, then the mean of $x_1 + 4, x_2 + 8, x_3 + 12 \ldots, x_{10} + 40$ is

(A) 34 (B) 32 (C) 42 (D) 38 (E) 40

32. A letter is taken at random from the word "STATISTICS" and another letter is taken at random from the word "ASSISTANT".The probability that they are same letters is

(A) $\frac{1}{45}$ (B) $\frac{13}{90}$ (C) $\frac{19}{90}$ (D) $\frac{5}{18}$ (E) $\frac{9}{10}$

Space for rough work

33. If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then

(A) $a^2 - b^2 + 2ac = 0$ (B) $(a - c)^2 = b^2 + c^2$

(C) $a^2 + b^2 - 2ac = 0$ (D) $a^2 + b^2 + 2ac = 0$

(E) $a + b + c = 0$

34. If the sides of a triangle are 4, 5 and 6 cms. Then the area of triangle is ______ sq.cms.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}\sqrt{7}$ (C) $\frac{4}{15}$ (D) $\frac{4}{15}\sqrt{7}$ (E) $\frac{15}{4}\sqrt{7}$

35. In a group of 6 boys and 4 girls, a team consisting of four children is formed such that the team has atleast one boy. The number of ways of forming a team like this is

(A) 159 (B) 209 (C) 200 (D) 240 (E) 212

36. A password is set with 3 distinct letters from the word LOGARITHMS. How many such passwords can be formed?

(A) 90 (B) 720 (C) 80 (D) 72 (E) 120

Space for rough work

37. If 5^{97} is divided by 52, the remainder obtained is

(A) 3 (B) 5 (C) 4 (D) 0 (E) 1

38. A quadratic equation $ax^2 + bx + c = 0$, with distinct coefficients is formed. If a, b, c are chosen from the numbers 2,3,5, then the probability that the equation has real roots is

(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) $\frac{2}{3}$

39. $\lim_{x\to\infty} \frac{3x^3+2x^2-7x+9}{4x^3+9x-2}$ is equal to

(A) $\frac{2}{9}$ (B) $\frac{1}{2}$ (C) $\frac{-9}{2}$ (D) $\frac{3}{4}$ (E) $\frac{9}{2}$

40. The minimum value of $f(x) = max\{x, 1 + x, 2 - x\}$ is

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 1 (D) 0 (E) 2

41. The equations of the asymptotes of the hyperbola $xy + 3x - 2y - 10 = 0$ are

(A) $x = -2\,, y = -3$ (B) $x = 2\,, y = -3$ (C) $x = 2\,, y = 3$

(D) $x = 4\,, y = 3$ (E) $x = 3\,, y = 4$

Space for rough work

42. If $f(x) = x^6 + 6^x$, then $f'(x)$ is equal to

(A) $12x$ (B) $x + 4$ (C) $6x^5 + 6^x \log(6)$

(D) $6x^5 + x6^{x-1}$ (E) x^6

43. The standard deviation of the data 6,5,9,13,12,8,10 is

(A) $\frac{\sqrt{52}}{7}$ (B) $\frac{52}{7}$ (C) $\frac{\sqrt{53}}{7}$ (D) $\frac{53}{7}$ (E) 6

44. $\lim_{x\to 0} \frac{1-\cos mx}{1-\cos nx} =$

(A) $\frac{m^2}{n^2}$ (B) $\frac{n^2}{m^2}$ (C) ∞ (D) $-\infty$ (E) 0

45. $\lim_{x\to 0} \frac{\sqrt{1+2x}-1}{x} =$

(A) 0 (B) -1 (C) $\frac{1}{2}$ (D) 1 (E) $\frac{-1}{2}$

46. Let f and g be differentiable functions such that $f(3) = 5$, $g(3) = 7$, $f'(3) = 13$, $g'(3) = 6$, $f'(7) = 2$ and $g'(7) = 0$. If $h(x) = (fog)(x)$, then $h'(3) =$

(A) 14 (B) 12 (C) 16 (D) 0 (E) 10

Space for rough work

47. $\frac{\sqrt{3}}{\sin(20^\circ)} - \frac{1}{\cos(20^\circ)} =$

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) 2 (D) 4 (E) 0

48. A Poisson variate X satisfies $P(X = 1) = P(X = 2)$. $P(X = 6)$ is equal to

(A) $\frac{4}{45}e^{-2}$ (B) $\frac{1}{45}e^{-1}$ (C) $\frac{1}{9}e^{-2}$ (D) $\frac{1}{4}e^{-2}$ (E) $\frac{1}{45}e^{-2}$

49. Let a and b be 2 consecutive integers selected from the first 20 natural numbers. The probability that $\sqrt{a^2 + b^2 + a^2b^2}$ is an odd positive integer is

(A) $\frac{9}{19}$ (B) $\frac{10}{19}$ (C) $\frac{13}{19}$ (D) 1 (E) 0

50. An ellipse of eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle. A point is chosen inside the circle at random. The probability that the point lies outside the ellipse is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{9}$ (D) $\frac{2}{9}$ (E) $\frac{1}{27}$

Space for rough work

51. If the vectors $4\hat{\imath} + 11\hat{\jmath} + m\hat{k}$, $7\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ and $\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$ are coplanar, then m is equal to

(A) 38 (B) 0 (C) 10 (D) -10 (E) 25

52. Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 7\hat{\imath} + 9\hat{\jmath} + 11\hat{k}$. Then the area of the parallelogram with diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is

(A) $4\sqrt{6}$ (B) $\frac{1}{2}\sqrt{21}$ (C) $\frac{\sqrt{6}}{2}$ (D) $\sqrt{6}$ (E) $\frac{1}{\sqrt{6}}$

53. If $|\vec{a}| = 3, |\vec{b}| = 1, |\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is equal to

(A) 13 (B) 26 (C) -29 (D) -13 (E) -26

54. If $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is equal to

(A) $\frac{\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{2}$ (D) 0 (E) π

Space for rough work

55. If the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$, $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} + \hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 9\hat{\jmath} + \mu\hat{k}$ are mutually orthogonal, then $\lambda + \mu$ is equal to

(A) 5 (B) -9 (C) -1 (D) 0 (E) -5

56. The solutions of $x^{2/5} + 3x^{1/5} - 4 = 0$ are

(A) 1,1024 (B) -1,1024 (C) 1,1031 (D) -1024,1 (E) -1,1031

57. If the equations $x^2 + ax + 1 = 0$ and $x^2 - x - a = 0$ have a real common root b, then the value of b is equal to

(A) 0 (B) 1 (C) -1 (D) 2 (E) 3

58. If $\sin\theta - \cos\theta = 1$, then the value of $\sin^3\theta - \cos^3\theta$ is equal to

(A) 1 (B) -1 (C) 0 (D) 2 (E) -2

59. Two dice of different colours are thrown at a time. The probability that the sum is either 7 or 11 is

(A) $\frac{7}{36}$ (B) $\frac{2}{9}$ (C) $\frac{2}{3}$ (D) $\frac{5}{9}$ (E) $\frac{6}{7}$

Space for rough work

60. $\frac{1}{9!}+\frac{1}{3!7!}+\frac{1}{5!5!}+\frac{1}{7!3!}+\frac{1}{9!}$ is equal to

(A) $\frac{2^9}{10!}$ (B) $\frac{2^{10}}{8!}$ (C) $\frac{2^{11}}{9!}$ (D) $\frac{2^{10}}{7!}$ (E) $\frac{2^8}{9!}$

61. The order and degree of the differential equation $(y''')^2+(y'')^3-(y')^4+y^5=0$ is

(A) 3 and 2 (B) 1 and 2 (C) 2 and 3 (D) 1 and 4 (E) 3 and 5

62. $\int_{-2}^{2}|x|dx$ is equal to

(A) 0 (B) 1 (C) 2 (D) 4 (E) $\frac{1}{2}$

63. $\int_{-1}^{0}\frac{dx}{x^2+2x+2}$ is equal to

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{-\pi}{4}$ (D) $\frac{\pi}{2}$ (E) $\frac{-\pi}{2}$

64. If $\int_{-1}^{4}f(x)dx=4$ and $\int_{2}^{4}(3-f(x))dx=7$, then $\int_{-1}^{2}f(x)dx$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Space for rough work

65. $\int \frac{xe^x}{(1+x)^2} dx =$

(A) $\frac{e^x}{1+x} + c$ (B) $\frac{e^x}{1+e^x} + c$ (C) $\frac{e^{2x}}{1+x} + c$ (D) $\frac{e^{-x}}{1+x} + c$ (E) $\frac{e^{-2x}}{1+x} + c$

66. The remainder when 2^{2000} is divided by 17 is

(A) 1 (B) 2 (C) 8 (D) 12 (E) 4

67. The coefficient of x^5 in the expansion of $(x+3)^8$ is

(A) 1542 (B) 1512 (C) 2512 (D) 2542 (E) 2452

68. The maximum value of $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ is

(A) 5 (B) 11 (C) 10 (D) -1 (E) 2

69. The area of the triangle in the complex plane formed by z, iz and $z + iz$ is

(A) $|z|$ (B) $|\bar{z}|^2$ (C) $\frac{1}{2}|z|^2$ (D) $\frac{1}{2}|z+iz|^2$ (E) $|z+iz|$

70. Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function. If f is even, then $f'(0)$ is equal to

(A) 1 (B) 2 (C) 0 (D) -1 (E) $\frac{1}{2}$

Space for rough work

71. The coordinate of the point dividing internally the line joining the points $(4,-2)$ and $(8,6)$ in the ratio $7:5$ is

(A) $(16,18)$ (B) $(18,16)$ (C) $\left(\frac{19}{3},\frac{8}{3}\right)$ (D) $\left(\frac{8}{3},\frac{19}{3}\right)$ (E) $(7,3)$

72. The area of the triangle formed by the points $(a,b+c),(b,c+a),(c,a+b)$ is

(A) abc (B) $a^2+b^2+c^2$ (C) $ab+bc+ca$

(D) 0 (E) $a(ab+bc+ca)$

73. If (x,y) is equidistant from $(a+b,b-a)$ and $(a-b,a+b)$, then

(A) $ax+by=0$ (B) $ax-by=0$ (C) $bx+ay=0$

(D) $bx-ay=0$ (E) $x=y$

74. The equation of the line passing through (a,b) and parallel to the line $\frac{x}{a}+\frac{y}{b}=1$ is

(A) $\frac{x}{a}+\frac{y}{b}=3$ (B) $\frac{x}{a}+\frac{y}{b}=2$ (C) $\frac{x}{a}+\frac{y}{b}=0$

(D) $\frac{x}{a}+\frac{y}{b}+2=0$ (E) $\frac{x}{a}+\frac{y}{b}=4$

Space for rough work

75. If the points $(2a, a)$, $(a, 2a)$ and (a, a) enclose a triangle of area 18 square units, then the centroid of the triangle is equal to

(A) (4,4) (B) (8,8) (C) $(-4,-4)$ (D) $(4\sqrt{2}, 4\sqrt{2})$ (E) (6,6)

76. The area of a triangle is 5 sq. units. Two of its vertices are (2,1) and $(3,-2)$. The third vertex lies on $y = x + 3$. The coordinates of the third vertex can be

(A) $\left(\frac{-3}{2}, \frac{-3}{2}\right)$ (B) $\left(\frac{3}{4}, \frac{-3}{2}\right)$ (C) $\left(\frac{7}{2}, \frac{13}{2}\right)$ (D) $\left(\frac{-1}{4}, \frac{11}{4}\right)$ (E) $\left(\frac{3}{2}, \frac{3}{2}\right)$

77. If $x^2 + y^2 + 2gx + 2fy + 1 = 0$ represents a pair of straight lines, then $f^2 + g^2$ is equal to

(A) 0 (B) 1 (C) 2 (D) 4 (E) 3

78. If θ is the angle between the pair of straight lines $x^2 - 5xy + 4y^2 + 3x - 4 = 0$, then $\tan^2\theta$ is equal to

(A) $\frac{9}{16}$ (B) $\frac{16}{25}$ (C) $\frac{9}{25}$ (D) $\frac{21}{25}$ (E) $\frac{25}{9}$

Space for rough work

79. If $3\hat{\imath}+2\hat{\jmath}-5\hat{k}=x(2\hat{\imath}-\hat{\jmath}+\hat{k})+y(\hat{\imath}+3\hat{\jmath}-2\hat{k})+z(-2\hat{\imath}+\hat{\jmath}-3\hat{k})$, then

(A) $x=1, y=2, z=3$
(B) $x=2, y=3, z=1$
(C) $x=3, y=1, z=2$
(D) $x=1, y=3, z=2$
(E) $x=2, y=2, z=3$

80. $\sin 15^\circ =$

(A) $\frac{\sqrt{3}-1}{2\sqrt{2}}$
(B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$
(C) $\frac{1-\sqrt{3}}{2\sqrt{2}}$
(D) $\frac{1+\sqrt{3}}{\sqrt{2}}$
(E) $\frac{-(1+\sqrt{3})}{2\sqrt{2}}$

81. If $\vec{a}$ and $\vec{b}=3\hat{\imath}+6\hat{\jmath}+6\hat{k}$ are collinear and $\vec{a}.\vec{b}=27$, then $\vec{a}$ is equal to

(A) $3(\hat{\imath}+\hat{\jmath}+\hat{k})$
(B) $\hat{\imath}+2\hat{\jmath}+2\hat{k}$
(C) $2\hat{\imath}+2\hat{\jmath}+2\hat{k}$
(D) $\hat{\imath}+3\hat{\jmath}+3\hat{k}$
(E) $\hat{\imath}-3\hat{\jmath}+2\hat{k}$

82. If $|\vec{a}|=13, |\vec{b}|=5$ and $\vec{a}.\vec{b}=30$, then $|\vec{a}\times\vec{b}|$ is equal to

(A) 30
(B) $\frac{30}{25}\sqrt{233}$
(C) $\frac{30}{33}\sqrt{193}$
(D) $\frac{65}{23}\sqrt{493}$
(E) $\frac{65}{13}\sqrt{133}$

83. If ${}^{56}P_{r+6}:{}^{54}P_{r+3}=30800:1$, then r is equal to

(A) 69
(B) 41
(C) 51
(D) 61
(E) 49

Space for rough work

84. Distance between two parallel lines $y = 2x + 4$ and $y = 2x - 1$ is

(A) 5 (B) $5\sqrt{5}$ (C) $\sqrt{5}$ (D) $\frac{1}{5}$ (E) $\frac{3}{\sqrt{5}}$

85. $({}^7C_0 + {}^7C_1) + ({}^7C_2 + {}^7C_3) + \ldots + ({}^7C_6 + {}^7C_7) =$

(A) $2^8 - 2$ (B) $2^7 - 1$ (C) 2^7 (D) $2^8 - 1$ (E) $2^7 - 2$

86. The coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$ is

(A) 17 (B) 19 (C) -17 (D) -19 (E) 20

87. The equation of the circle with centre (2,2) which passes through (4,5) is

(A) $x^2 + y^2 - 4x + 4y - 77 = 0$ (B) $x^2 + y^2 - 4x - 4y - 5 = 0$

(C) $x^2 + y^2 + 2x + 2y - 59 = 0$ (D) $x^2 + y^2 - 2x - 2y - 23 = 0$

(E) $x^2 + y^2 + 4x - 2y - 26 = 0$

88. The point in the *xy*-plane which is equidistant from (2,0,3), (0,3,2) and (0,0,1) is

(A) (1,2,3) (B) (−3,2,0) (C) (3, −2,0) (D) (3,2,0) (E) (3,2,1)

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89. Let $f: \mathbb{N} \to \mathbb{N}$ be such that $f(1) = 2$ and $f(x + y) = f(x)f(y)$ for all natural numbers x and y. If $\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$, then a is equal to

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

90. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then $n =$

(A) 3 (B) 4 (C) 8 (D) 9 (E) 10

91. Let $f: (-1,1) \to (-1,1)$ be continuous, $f(x) = f(x^2)$ for all $x \in (-1,1)$ and $f(0) = \frac{1}{2}$. Then the value of $4f\left(\frac{1}{4}\right)$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

92. $\lim_{x\to\infty} \sqrt{x^2 + 1} - \sqrt{x^2 - 1} =$

(A) -1 (B) 1 (C) 0 (D) 2 (E) 4

Space for rough work

93. If f is differentiable at $x = 1$ and $\lim_{h \to 0} \frac{1}{h} f(1+h) = 5$, then $f'(1) =$

(A) 0 (B) 1 (C) 3 (D) 4 (E) 5

94. The maximum value of the function $2x^3 - 15x^2 + 36x + 4$ is attained at

(A) 0 (B) 3 (C) 4 (D) 2 (E) 5

95. If $\int f(x) \cos x \, dx = \frac{1}{2}\{f(x)\}^2 + c$, then $f\left(\frac{\pi}{2}\right)$ is

(A) c (B) $\frac{\pi}{2} + c$ (C) $c + 1$ (D) $2\pi + c$ (E) $c + 2$

96. $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x} dx =$

(A) $\pi(\sqrt{2} - 1)$ (B) $\pi(\sqrt{2} + 1)$ (C) $2\pi(\sqrt{2} - 1)$

(D) $2\pi(\sqrt{2} + 1)$ (E) $\frac{\pi}{\sqrt{2}+1}$

Space for rough work

97. $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}} dx =$

(A) 2 (B) π (C) $\frac{\pi}{4}$ (D) 2π (E) 0

98. $\lim_{x\to 0}\left(\frac{\int_0^{x^2} \sin\sqrt{t}dt}{x^2}\right) =$

(A) $\frac{2}{3}$ (B) $\frac{2}{9}$ (C) $\frac{1}{3}$ (D) 0 (E) $\frac{1}{6}$

99. The area bounded by $y = \sin^2 x, x = \frac{\pi}{2}$ and $x = \pi$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{16}$ (E) 2π

100. The differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constants is

(A) $y'' - 2y' + 2y = 0$ (B) $y'' + 2y' - 2y = 0$ (C) $y'' + y'^2 + y = 0$
(D) $y'' + 2y' - y = 0$ (E) $y'' - 2y' - 2y = 0$

Space for rough work

101. The real part of $\left(i - \sqrt{3}\right)^{13}$ is

(A) 2^{-10} (B) 2^{12} (C) 2^{-12} (D) -2^{-12} (E) 2^{10}

102. $\lim_{x\to 0}\frac{1+x-e^x}{x^2} =$

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) 1 (D) -1 (E) 0

103. $\int\frac{(\sin x+\cos x)(2-\sin 2x)}{\sin^2 2x}dx =$

(A) $\frac{\sin x+\cos x}{\sin 2x}+c$ (B) $\frac{\sin x-\cos x}{\sin 2x}+c$ (C) $\frac{\sin x}{\sin x+\cos x}+c$

(D) $\frac{\sin x}{\sin x-\cos x}+c$ (E) $\frac{\sin x-\cos x}{\sin x+\cos x}+c$

104. A plane is at a distance of 5 units from the origin; and perpendicular to the vector $2\hat{\imath}+\hat{\jmath}+2\hat{k}$. The equation of the plane is

(A) $\vec{r}.\left(2\hat{\imath}+\hat{\jmath}-2\hat{k}\right)=15$ (B) $\vec{r}.\left(2\hat{\imath}+\hat{\jmath}-\hat{k}\right)=15$

(C) $\vec{r}.\left(2\hat{\imath}+\hat{\jmath}+2\hat{k}\right)=15$ (D) $\vec{r}.\left(\hat{\imath}+\hat{\jmath}+2\hat{k}\right)=15$

(E) $\vec{r}.\left(\widehat{2\imath}-\hat{\jmath}+2\hat{k}\right)=15$

Space for rough work

105. $\frac{\sin A - \sin B}{\cos A + \cos B}$ is equal to

(A) $\sin\left(\frac{A+B}{2}\right)$ (B) $2\tan(A+B)$ (C) $\cot\left(\frac{A-B}{2}\right)$

(D) $\tan\left(\frac{A-B}{2}\right)$ (E) $2\cot(A+B)$

106. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2} =$

(A) x (B) $-16x$ (C) $15x$ (D) $16x$ (E) $-15x$

107. The arithmetic mean of ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ is

(A) $\frac{2^n}{n+1}$ (B) $\frac{2^n}{n}$ (C) $\frac{2^{n-1}}{n+1}$ (D) $\frac{2^{n-1}}{n}$ (E) $\frac{2^{n+1}}{n}$

108. The variance of first 20 natural numbers is

(A) $\frac{399}{2}$ (B) $\frac{379}{12}$ (C) $\frac{133}{2}$ (D) $\frac{133}{4}$ (E) $\frac{169}{2}$

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109. If S is a set with 10 elements and $A = \{(x, y): x, y \in S, x \neq y\}$, then the number of elements in A is

(A) 100 (B) 90 (C) 80 (D) 150 (E) 45

110. A coin is tossed and a die is rolled. The probability that the coin shows head and the die shows 3 is

(A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{1}{9}$ (D) $\frac{11}{12}$ (E) $\frac{1}{11}$

111. If $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$, then the sum of all the diagonal entries of A^{-1} is

(A) 2 (B) 3 (C) -3 (D) -4 (E) 4

112. Let $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$. If $x = -9$ is a root of $f(x) = 0$, then the other roots are

(A) 2 and 7 (B) 3 and 6 (C) 7 and 3 (D) 6 and 2 (E) 6 and 7

Space for rough work

113. If $[1 \quad x \quad 1]\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = 0$, then x can be

(A) -2 (B) 2 (C) 14 (D) -14 (E) 0

114. If $A = \begin{bmatrix}2x & 0\\ x & x\end{bmatrix}$ and $A^{-1} = \begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$, then $x =$

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) 3 (E) 0

115. If $\begin{vmatrix}x & 2 & x\\ x^2 & x & 6\\ x & x & 6\end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$, then $5a + 4b + 3c + 2d + e$ is equal to

(A) 11 (B) -11 (C) 12 (D) -12 (E) 13

Space for rough work

116. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} =$

(A) 1 (B) 0 (C) $(1-a)(1-b)(1-c)$

(D) $a+b+c$ (E) $2(a+b+c)$

117. If $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$, then $f(50) =$

(A) 0 (B) 2 (C) 4 (D) 1 (E) 3

118. If $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$, then $\int_0^{\pi/2} \Delta(x)dx =$

(A) $\frac{-1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) -1 (E) 0

Space for rough work

119. The equation of the plane passing through the points $(1,2,3), (-1,4,2)$ and $(3,1,1)$ is

(A) $5x + y + 12z - 23 = 0$

(B) $5x + 6y + 2z = 23$

(C) $x + 6y + 2z = 13$

(D) $x + y + z = 13$

(E) $2x + 6y + 5z = 7$

120. In an arithmetic progression, if the k^{th} term is $5k + 1$, then the sum of first 100 terms is

(A) $50(507)$ (B) $51(506)$ (C) $50(506)$ (D) $51(507)$ (E) $52(506)$

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