

MATHEMATICS

Paper-II

Time: 3 Hours

Full Marks: 100

Instructions : (1) Answer any five questions.

(2) The figures in the right-hand margin indicate full marks for the questions.

1. (a) Describe the principle of mathematical induction. Prove by the principle of mathematical induction that

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} \quad 10$$

- (b) Write about the following briefly : 2×5=10

- (i) The relation between Permutation and Combination.
- (ii) The r th term ($r \leq n$) in the binomial expansion of $(a - b)^n$.
- (iii) The sum up to n th term of a geometric progression with initial term a and the geometric ratio r .
- (iv) The volume of a right circular cone with base radius r and height h .
- (v) The relation between the three cube roots of unity.

2. Write brief notes on the following : 4×5=20

- (a) Linear inequalities in one variable
- (b) Mean, Median and Mode of a distribution
- (c) The order and the degree of a differential equation
- (d) Invertible matrices
- (e) Conic section in coordinate geometry

3. Answer the following : 4×5=20

- (a) Find the term absent in x in the binomial expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$.
- (b) Calculate the area of the parallelogram whose adjacent sides are defined by the Cartesian vectors $2\hat{i} + 3\hat{j}$ and $-\hat{i} + \hat{j}$.
- (c) Construct the adjoint of matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & -4 \\ -1 & 5 & 2 \end{pmatrix}$$

(d) Solve the ordinary differential equation $x^2 \frac{dy}{dx} + y = 1$.

(e) Integrate

$$\int \sqrt{\frac{1+x}{1-x}} dx$$

4. (a) Explain the matrix inversion method of solving a system of linear simultaneous equations. Solve the following system of equations by matrix inversion method : $3+7=10$

$$x + y + z = 9$$

$$2x + 5y + 7z = 52$$

$$2x + y - z = 0$$

(b) Describe the properties of the scalar triple product of vectors. Find

$$\vec{A} \cdot (\vec{B} \times \vec{C}) \text{ if } \vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \vec{C} = 3\hat{i} + \hat{j} + 2\hat{k} \quad 10$$

5. Establish that a homogeneous equation of second degree of the type $ax^2 + 2hxy + by^2 = 0$ which represents a pair of straight lines passing through the origin. Find the expression for the angle between the lines. Show that the pair of straight lines represented by the equation $3x^2 + 2xy - 3y^2 = 0$ are perpendicular to each other. 7+7+6=20

6. In a certain bacterial culture, the number of bacteria is found to be increasing at the rate proportional to the number of bacteria present at the moment in consideration. Write down the differential equation for the problem. If the number of bacteria doubles every 3 hours, calculate the number of bacteria after 12 hours, when 100 bacteria were present initially. 5+15=20

7. A competition was participated by 4 boys and 2 girls from school A, 2 boys and 2 girls from school B, and 3 boys and 3 girls from school C. It was known that one girl was the winner of the competition. Calculate the probability that the girl was from school A. 20

8. (a) If $i = \sqrt{-1}$, then find the value of $i + i^2 + i^3 + \dots + i^{99}$. 6

(b) Given $x + y = 5$, find the minimum value of $x^2 + y^2$. 8

(c) Find the area under the curve $y = x^2 + x + 4$ and the x -axis bounded by $x = 0$ and $x = 3$. 6