

**STATISTICS**  
**(Subject Code-77)**

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**Unit-1 : Probability Theory :**

Random experiment: Trial, sample point, sample space, definitions of equally likely, mutually exclusive and exhaustive events, events class, definition of probability, classical and relative frequency approach to probability, axiomatic approach to probability and its properties, merits and demerits of these approaches, total and compound probability theorems, conditional probability, independence of events, Bayes' theorem and its applications.

**Unit-2 : Distribution Theory :**

Random Variable: Concept of discrete random variable, probability mass function and distribution function, joint probability mass function of several discrete random variables, marginal and conditional probability mass functions. Continuous random variable: Probability density function, distribution function, joint density function of two continuous variables, marginal and conditional probability density functions, independence of random variables. Mathematical expectation, moments, factorial moments, moment generating function of a random variable, their properties and uses, probability generating function, Chebyshev's inequality and its applications, basic ideas of convergence in probability and convergence in distribution, Markov's inequality, functions of random variables. Bernoulli distribution, binomial distribution, Poisson distribution, geometric distribution, negative binomial distribution, hyper geometric distribution, multinomial distribution, uniform distribution, normal distribution and its relationship with the binomial and Poisson distribution, Cauchy distribution, bivariate normal distribution and its marginal and conditional distributions.

**Unit-3 : Statistical Methods :**

Concept of Statistical population and sample, classification of data, quantitative and qualitative data, ordinal and nominal data, time series and cross-sectional data, multivariate data.

Construction of tables (with one or more factors), diagrammatic and graphical representation of grouped data, frequency and cumulative frequency distribution and their applications, histogram, frequency polygon, ogives, stem and leaf charts, box plot.

Concept of central tendency and its measures, partition values, dispersion and relative dispersion, moments, Sheppard's correction for moments (without derivation), skewness, kurtosis and their measures.

Bivariate data: Scatter diagram, product moment correlation coefficient and its properties, coefficient of determination, correlation ratio, interclass correlation, and concept of error in regression, principle of least square, fitting of linear regression and related results, rank correlation.

Partial and multiple correlation in three variables, their measures and related results.

Theory of attributes: Independence and Association of attributes, various measures of association for two way classified data.

**Unit-4 : Statistical Quality Control and Time Series :**

Statistical process and product control: Quality of a product, need for quality control, basic concept of process control, process capability and product control, general theory of control charts, causes of variation in quality, control limits, sub grouping summary of

out of control criteria. Charts for attributes: p chart, np chart, c-chart, Charts for variables:  $(\bar{X}, R)$ ,  $(\bar{X}, \sigma)$  charts.

Principle of acceptance sampling-problem of lot acceptance, stipulation of good and bad lots, producers and consumers risks, single and double sampling plans, their OC functions, concepts of AQL, LTPD, AOQL, average amount of inspection and ASN function.

Measurement of Fertility: Crude birth rate, general fertility rate, age specific birth rate, total fertility rate, gross reproduction rate, net reproduction rate, logistic model for population projection.

Time Series Analysis: Economic time series, different components, illustration, additive and multiplicative models, determination of trend, seasonal and cyclical fluctuations.

#### **Unit-5 : Demography and Index Number :**

Measurement of Mortality and Life Table: Crude death rate, Standardized death rates, Age-specific death rates, Infant Mortality rate, Death rate by cause, Complete and abridged life table and its main features, Uses of life table.

Index Numbers: Price relatives and quantity or volume relatives, Link and chain relatives composition of index numbers; Laspeyre's, Paasche's, Marshal-Edgeworth's and Fisher's index numbers; chain base index number, tests for index number, cost of living index number.

#### **Unit-6 : Statistical Inference :**

Sampling distribution of a statistic, Derivation of  $\chi^2$ , t, F and Z distributions, Beta Gamma and Laplace densities.

Point estimation: properties of estimators, mean square and minimum mean square error estimator, unbiasedness and minimum variance unbiased estimator, Cramer-Rao lower bound, amount of information, consistency of estimators and sufficient conditions for consistency, relative efficiency of an estimator, asymptotic efficiency, sufficiency, factorization theorem (without proof), concept of complete sufficient statistics, Rao-Blackwell theorem. Completeness and sufficiency, Lehman Scheffe theorem, one parameter exponential family and its completeness, Cramer-Rao inequality, Best linear unbiased estimator.

Methods of estimation: moments, maximum likelihood, minimum chi-square, least square with examples, BAN and CAN estimators, point estimates of measures of location, dispersion, regression, correlation and other useful parameters.

Concepts of confidence interval and confidence coefficient, confidence intervals for the parameters of univariate normal, two independent normal distributions and exponential distributions.

Statistical hypotheses, critical region, size and power of a test, most powerful test, randomized and non-randomized test, Neyman-Pearson lemma and its applications, uniformly most powerful unbiased test, power likelihood ratio test and its applications, functions of UMP with simple illustration.

Applications of  $\chi^2$ , t, F and z distributions in tests of significance.

Likelihood ratio test, Unbiased test, Neyman Pearson Lemma for randomized tests, randomized test for binomial and Poisson distribution,  $\chi^2$  –test of goodness of fit, Test of equality of several variances, Significance test for correlation coefficient.

#### **Unit-7 : Sampling Theory :**

Concept of population and sample, need for sampling, complete enumeration versus sampling, Basic concepts in sampling, sampling and Non-sampling errors.

Simple random sampling with and without replacement, estimation of population mean, population proportion and their standard errors. Stratified random sampling, proportional and optimum allocation, comparison with simple random sampling for fixed sample size. Post stratification, Double sampling with post stratification.

Ratio, product and regression methods of estimation, estimation of population mean, evaluation of bias and variance to the first order of approximation, comparison with simple random sampling.

Systematic sampling, Cluster sampling with equal clusters, two stage sampling.

Unequal probability sampling: PPSWR/PPSWOR methods of sample selection (including cumulative total method and Lahiri's scheme). Comparison of SRSWR and PPSWR schemes. Ordered estimators of Des Raj and Murthy (for  $n=2$ ). Construction of unordered estimators from ordered estimators. Horvitz Thompson's estimator of a finite population total/mean. Expression for variance of Horvitz Thompson's estimator and unbiased estimator of the variance, Yates-Grundy modification.

Kinds of non-sampling errors with special reference to non-response problems. Hansen and Hurwitz estimator for population mean. Concept of randomized response and some well-known randomized response techniques for sensitive characteristics.

#### **Unit-8 : Linear Estimation and Design of Experiments :**

General linear model, assumptions, estimation of parameters by least squares, estimable functions, error and estimation space, Gauss-Markov theorem.

Distribution of quadratic form and its application in analysis of variance model, Estimable linear hypothesis, generalized F and t tests.

One-way ANOVA, two-way ANOVA with single observation per cell and equal number of observations per cell, Tukey's test.

Randomization, Replication, Local Control, Completely Randomized Design (CRD), Randomized Block Design (RBD), Latin square design (LSD), Missing plot technique.

General block design and its information matrix (C). Criteria for connectedness, balanced and orthogonality, Intra-block analysis, Incomplete block design: Balanced Incomplete Block Design (BIBD).

General factorial experiments, factorial effects, study of  $2^n$  and  $3^n$  factorial experiments in randomized blocks, complete and partial confounding, construction of confounded factorial experiments, split plot experiment.

#### **Unit-9 : Multivariate Analysis :**

Multivariate normal distribution (MND), normal generating function and Characteristic function, marginal and conditional distributions, multiple and partial correlation coefficients for MND.

Maximum likelihood estimators of the mean vector and covariance matrix, Distribution of sample mean vector, null distribution of sample correlation coefficient, sample multiple and partial correlation coefficients and their null sampling distributions, distribution of sample regression coefficient.

Null distribution and non-null distribution of Hotelling's  $T^2$  statistic, Application in tests for mean vector of one and more multivariate normal populations and for equality of the components of a mean vector in a multivariate normal population and their applications, Mahalanobis'  $D^2$ , Wishart distribution and its properties.

Classification and discrimination procedures for discrimination between two multivariate normal distributions, populations-sample discriminant function, and test associated with discriminant functions, probabilities of misclassification and their

estimation, classification into more than two multivariate normal populations, Fisher-Behrens Problem.

**Unit-10 : Advance Probability Theory :**

Classes of sets, fields, sigma fields, minimal sigma field, Borel sigma field, sequence of sets, *lim sup* and *lim inf* of a sequence of sets, measure, probability measure, properties of measure, Caratheodory, Lebesgue and Lebesgue-Stieltjes measures.

Measurable functions, random variables, sequence of random variables, Integration of a measurable function with respect to a measure, monotone convergence theorem, Fatou's lemma, dominated convergence theorem. Characteristic function, uniqueness theorem, Levy's continuity theorem (statement only), Convergence in distribution, Convergence in probability, almost sure convergence.

Decomposition of distribution functions in purely discrete, absolutely continuous and singular components. Holder's inequality, Minkowski inequality, Lyapunov, Kolmogorov's inequality.

Weak (WLLN) and Strong (SLLN) law of Large Numbers, Khintchine Theorem and Kolmogorov Strong Law of Large Numbers, Borel zero-one Law, Borel-Cantelli lemma.

Weak and complete convergence of sequence of distribution functions, Weak compactness Theorem, Helly-Bray Lemma & Theorem Characteristics functions, Inversion Theorem

One dimensional Central Limit Theorems: Lindberg- Levy for *i.i.d.* random variables.